Application of Digital Annealer for Faster Combinatorial Optimization

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There are combinatorial optimization problems in our society—selecting the best option from combinations of various factors, such as finding the best procedures in disaster recovery efforts and optimizing investment portfolios. In combinatorial optimization problems, the number of combinations increases exponentially as the number of factors increases, which makes it extremely time-consuming for general-purpose computers to solve certain problems within a realistic time frame. Research and development in quantum computing is underway in order to solve combinatorial optimization problems quickly. However, the current state of quantum computing is limited in terms of stable operation and the size of problems it can handle. Furthermore, quantum computing requires the conversion of a combinatorial optimization problem into an Ising model to solve it. Against this background, Fujitsu launched its Digital Annealer Service in May 2018. This is a new architecture inspired by quantum computing. This paper explains the technology to employ Digital Annealer to solve customers' real combinatorial optimization problems, namely, the formulation of real problems and the conversion to quadratic unconstrained binary optimization (QUBO). It also describes the initiatives at Fujitsu to leverage Digital Annealer in creating a new global computing market.

1. Introduction

Computers came into being in the mid-20th century, and their rapid progress has helped to enrich our lives. General-purpose computers underwent miniaturization in order to achieve faster computing speeds, as predicted by Moore's law (the density of semiconductors doubles every 18 months). However, this progress is said to have come to its own limit recently.[1] Given this, there is a risk that computers will soon be overloaded with the growing volume of data created on a daily basis and increasingly complicated tasks. Therefore, there is a need for a completely new concept regarding computers that is different from the simple approach of trying to increase computing speeds. One such new concept that is garnering attention is quantum computing, and there are fast-developing worldwide efforts to bring this to practical application.

There are two approaches in quantum computing: quantum gates and Ising machines. Quantum computing using quantum gates is under development and is expected to take decades to realize. On the other hand, the Ising machine type has some commercialized examples through one of its variants, the annealing model. However, there are two major challenges with the Ising machine. One is that it is difficult to maintain stable quantum bits (qbits), which are susceptible to noise and destabilized easily. Another difficulty is that applicable problems must be rather small in size due to the small number of qbits and inter-qbit connections.[2]

Our customers need better and more efficient computer performance to process increasing amounts of data and various increasingly complicated tasks. Therefore, their businesses cannot wait until quantum computing becomes commercialized.

In order to respond to such demands, Fujitsu has thus developed Digital Annealer, a new computer architecture that incorporates the merits of both quantum and general-purpose computers. Digital Annealer realizes annealing on a conventional digital circuit, and specializes in rapidly solving combinatorial optimization problems.[3]

Digital Annealer can solve the two challenges of quantum computing stated above. Firstly, the digital circuit is resistant against noise, thus ensuring stable operations. Secondly, each bit is fully connected at the 1,024-bit scale, enabling Digital Annealer to handle large-scale problems.

Fujitsu has deployed Digital Annealer to solve problems in a wide range of customer environments, from chemical and finance to logistics.[4)]

In this paper, we explain our efforts to identify specific problems in combinatorial optimization and conversion to quadratic unconstrained binary optimization (QUBO), with accounts of specific cases.

2. Architecture of Digital Annealer service

The Digital Annealer service[5)] offers solutions to combinatorial optimization problems by deploying Fujitsu Laboratories' hardware with software developed by 1QB Information Technologies Inc. (hereafter, 1QBit).[6), 7)]

The hardware has been developed by Fujitsu Laboratories[8)] specifically for solving evaluation function QUBO. It employs mainly two approaches to obtain an optimal solution quickly, namely, a batch search method that accelerates computational speed, and the search method that enhances the probability of escaping from local optima. These approaches enable faster computation of QUBO than conventional methods such as simulated annealing.[9)] At the same time, by lowering the probability of remaining with the local optima, the probability of obtaining an optimal solution is enhanced.[3)]

1QBit offers a library in the form of a web application programming interface (web-API) that facilitates the conversion of formulated expressions into QUBO style or Ising machine style.

3. Ising model and QUBO

The Ising model was created as a simplified model to handle the phase transition of ferromagnetic materials.[9)]

With the Ising model, the interaction between spins σ_i with the values +1 and −1 determines the energy. The evaluation function $E(\sigma)$ for the entire spin $\sigma=(\sigma_1, \sigma_2, \ldots)$ can be expressed as follows:

$$E(\sigma) = -\frac{1}{2}\underset{\text{Interaction member}}{\sum_i\sum_j J_{ij}\sigma_i\sigma_j} - \underset{\text{Static magnetic field member}}{\sum_i h_i\sigma_i}.$$

If, by converting spin σ_i by the equation $x_i = \frac{\sigma_i+1}{2} \in \{0, 1\}$, the evaluation function $E(x)$ of all variables $x=(x_1, x_2, \ldots)$ is expressed using x_i, this will take the QUBO style. Digital Annealer takes the QUBO style for the reasons that binary variables are highly compatible with digital circuits and that it is easy to lower the degree of the formula.

4. Technology to formulate problems

In order to obtain an optimal solution, it is necessary to take the following five steps from extracting issues to be solved as a combinatorial optimization problem and calculating a solution with Digital Annealer (**Figure 1**).

- Step 1: Extract issues

Conduct interviews with customers to extract issues they face and identify combinatorial optimization problems to be solved.

- Step 2: Conversion to combinatorial optimization problems

Identify problems that can be solved as combinatorial optimization problems from the extracted issues.

- Step 3: Formulation

Determine if any patterns of existing well-known formulated combinatorial optimization problems (traveling salesman problem, maximum cut problem, bin packing problem, minimum vertex cover problem, etc.) can be applied to the identified problem. If the existing problems cannot be applied, the problem is formulated as a new combinatorial optimization problem. At this step, it is necessary to define the problem by expression, such as with objective functions and constraint conditions.

- Step 4: Conversion to QUBO

Convert the formulas into QUBO.

- Step 5: Computation for optimal solutions

The obtained QUBO will be converted to a style compatible with Digital Annealer automatically by 1QBit software. The optimal solution is obtained by solving the QUBO with Digital Annealer. Finally, the best combination thus obtained is evaluated to determine whether or not to adopt it as the solution for the problem.

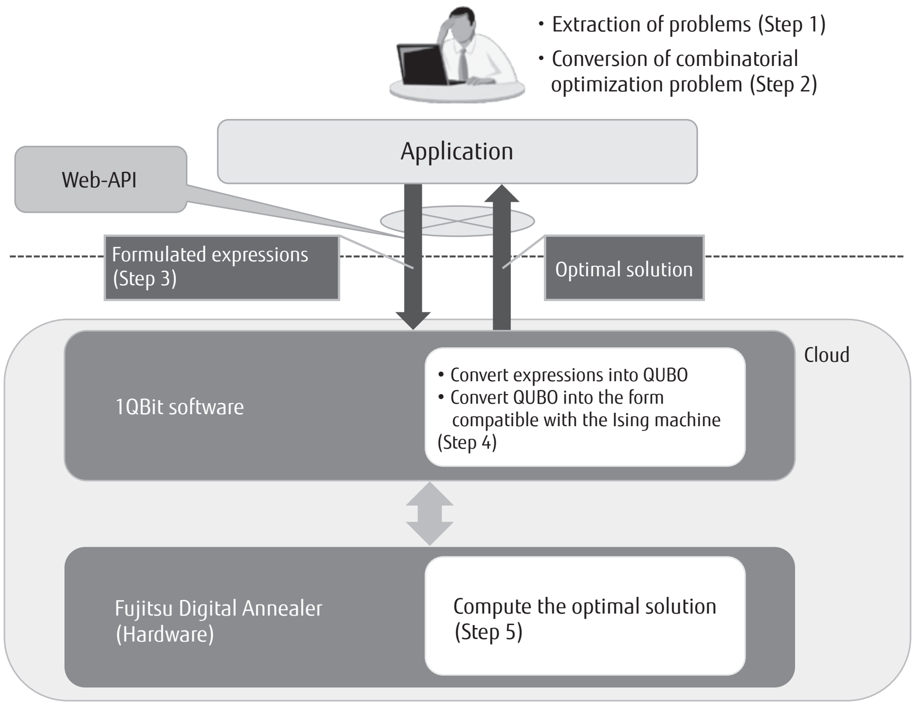


Figure 1
Digital Annealer service.

In the following sections, we describe examples of Steps 2 and 4 as key stages for obtaining optimal solutions.

5. Application case 1: Placement and layout optimization of parts shelves in a warehouse

In this section, we present an example of Step 2 to identify combinatorial optimization problems from extracted issues at Fujitsu IT Products Ltd. and their optimization problem concerning placement and layout of the parts shelves in the warehouse.

Many models are produced in small quantities at the plant. Regardless of the type of products, as many common parts are often used in production, parts are not arranged for each product. When manufacturing a product, workers walk through the shelves to collect the necessary parts. However, they rely on their own know-how to find efficient routes to collect parts, which may not necessarily be the most optimal route. As workers have been deciding which parts to put on which shelves, it required a long time to consider. Against this background, they have been working on addressing the following two issues at this plant.

- Issue 1: To specify optimal routes to collect parts without relying on workers' know-how.
- Issue 2: To easily calculate the optimal placement of each shelf.

In the following description, as an example, we explain converting Issue 2, described above, into a combinatorial optimization problem through the following procedure.

The problem of optimizing the placement of the shelves can be solved by placing parts to be collected at the same time with high frequency (strongly correlated) on a nearby shelf and placing parts to be collected at the same time with low frequency (weakly correlated) on a distant shelf. Hierarchical clustering can be applied to this optimization[10] (**Figure 2**).

Hierarchical clustering is a method to visualize the data correlations. There are two approaches to this method. One is the agglomerative method, in which clusters are sequentially merged from a state where data is divided into individual small clusters to generate clusters hierarchy. The other is the divisive method, which starts from the state of one cluster and recursively divides the target set repeatedly. Of these, it is the divisive method that can extract problems solvable

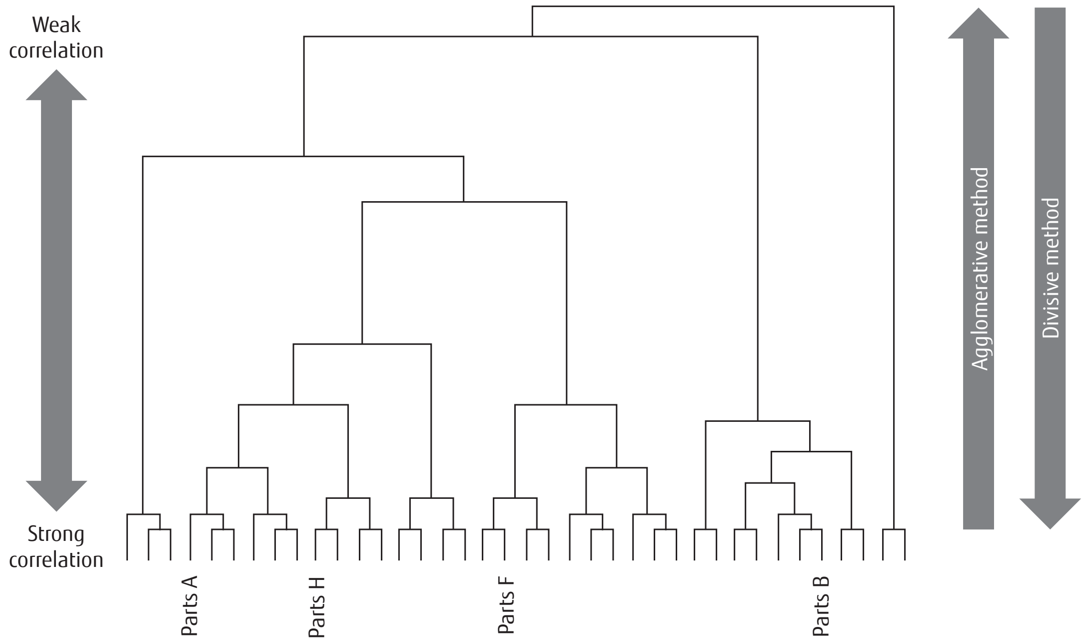


Figure 2
Hierarchical clustering.

through the combinatorial optimization problem using Digital Annealer.

In order to perform clustering according to the divisive method, the "maximum cut problem" can be applied, one of the combinatorial optimization methods that can divide parts into strongly correlated and weakly correlated parts. This method has been applied to a financial case by 1Qbit, and there were no difficulties in applying it to this case. Practically speaking, we use a period-worth of ledger data employed in parts collection and create a weighted undirected graph (comprising nodes and edges, with weights assigned to the edges).

Consider a graph in which the parts are nodes and the frequency of simultaneous collection is the edge weight. The maximum cut problem of the graph seeks to maximize the weight of the remaining edge when dividing the graph into two. In this case, we left as many sets of parts with high frequency of simultaneous collection as possible and group the parts into two (**Figure 3**). Grouped parts are further separated into two groups, and grouping is repeated until the parts become the minimum unit.

As a result, by clustering according to the strength of the correlation, we could propose optimized placement of parts and shelf layout. When we applied the

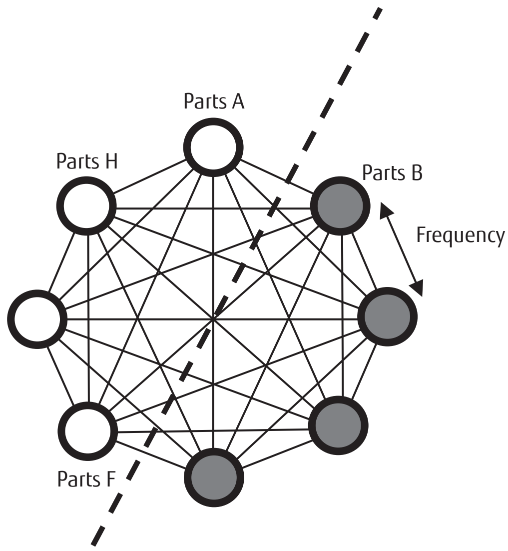


Figure 3
Production parts correlations.

optimized layout of parts and shelf layout, we obtained the verification result that the travel distance per month of the workers was shortened by 23.6% (the total moving distance had been shortened from 25,001 m to 19,080 m). Additionally, when we applied the clustering method to Issue 1 to optimize the work route, we obtained the result that the travel distance per month was further reduced by 21.7% (the total travel distance had been reduced from 19,080 m to 14,926 m).

In this way, extraction of the solvable problem obtained by solving the combinatorial optimization problem can be realized with the idea of modeling and deforming the problem in order and applying it to the known case.

6. Application case 2: Assignment of workers

In this section, as an example of Step 4 of converting formulated mathematical equations to QUBO, we introduce worker placement optimization at a Japanese logistics company. Since we could not apply this case to any formulated combinatorial optimization problems, we applied original model and formulation to this case.

For a particular work shift, responsible persons had been assigning workers, and the decisions had taken a lot of time. Moreover, their assignments were not necessarily optimal. This problem can be formulated as a mathematical equation in the following procedure and represented by QUBO.

Here, we explain an example of optimizing worker assignment when assigning shifts for one week to 34 workers. **Figure 4** shows the number of workers assigned by hour. A worker is represented as the variable a (34 persons from 0 to 33) and a shift (4 shifts/day from 0 to 3) as the variable t. Furthermore, the variable d represents the day of the week (7 days from 0 to 6).

First, we define an objective function as the total number of workers in a week. Furthermore, we formulate the problem as a combinatorial optimization problem to minimize the objective function (i.e., to maximize the number of workers who work zero days in

	d=0				d=1				…	d=6			
a \ t	0	1	2	3	0	1	2	3	…	0	1	2	3
0	0	0	1	0	0	0	0	1		1	0	0	0
1	1	0	0	0	1	0	0	0		0	1	0	0
2	0	0	0	1	0	0	0	0		0	0	0	1
3	0	1	0	0	0	0	1	0	…	0	0	0	0
4	0	0	0	0	0	0	0	0		0	0	0	0
⋮													
32	1	0	0	0	0	1	0	0		0	0	1	0
33	0	1	0	0	0	0	0	0		1	0	0	0

Figure 4
Workforce assignment.

a week). This problem is formulated as the expression:

Minimize $\Sigma_a f(a)$,

$$f(a) = \begin{cases} 1 \ \ if\, \Sigma_{d,t}\, x_{a,d,t} > 0, \\ 0 \ \ otherwise, \end{cases} \tag{1}$$

where $x_{a,d,t}$ represents work assignment for a worker a on a day of the week d and a shift t. If the worker is assigned work, $x_{a,d,t}$ takes 1, otherwise 0. Therefore, $\sum_{d,t} x_{a,d,t}$ represents the number of working days in a week for a certain worker a. $f(a)$ takes 1 if a worker a works at least one day in a week, otherwise 0. Furthermore, $\sum_a f(a)$ represents the total number of workers who work at least one day in a week.

In addition, although there are multiple constraints when solving this case, we explain two representative constraints below.

The first constraint condition is adding the difference between the amount of work (S^d) that needs to be performed on a day of the week d by all workers and the amount of work ($\sum_{t,a} s^a\, x_{a,d,t}$) actually allocated to the workers minimized as:

$$\min \left| \sum_{t,a} s^a\, x_{a,d,t} - S^d \right| \ (d=0,1,2,\ldots,6), \tag{2}$$

where s^a is the skill of the worker a, which means the volume of work that can be performed by the worker a in a shift on a given day.

The second constraint condition is that each worker takes two or more days off per week (work is performed on five or less days in a week). This is formulated as the following inequation:

$$\sum_{d,t} x_{a,d,t} \leq 5 \ \ (a=0,1,2,\ldots,33). \tag{3}$$

To summarize the above, the optimization problem of worker assignment is formulated as follows:

Objective function:

Minimize $\sum_a f(a)$,

$$f(a) = \begin{cases} 1 \ \ if\, \Sigma_{d,t}\, x_{a,d,t} > 0, \\ 0 \ \ otherwise, \end{cases}$$

Constraints:

$$\min \left| \sum_{t,a} s^a\, x_{a,d,t} - S^d \right| \ \ (d=0,1,\ldots,6),$$

$$\sum_{d,t} x_{a,d,t} \leq 5 \ \ (a=0,1,2,\ldots,33).$$

Next, we explain a method of converting this formulated optimization problem to QUBO in order to solve by Digital Annealer. In QUBO, we add the objective function and the constraint conditions and minimize (or maximize) it.

Expression (2) first deforms to take the minimum value when there is absolutely no difference between the work volume S^d that needs to be done on a single day and the work volume actually allocated. The transformed expression is as follows:

$$\sum_d\left(\sum_{t,a} s^a x_{a,d,t} - S^d\right)^2 . \tag{4}$$

In the conversion of inequation (3) to QUBO, the minimum value is taken when the number of working days of workers is 0 days, or 2 to 5 days. We also use two supplementary variables $y_{a,0}$ and $y_{a,1}$ to give a range of working days as:

$$\sum_a\left(\sum_{d,t}x_{a,d,t}+2y_{a,0}+5y_{a,1}-4\right)\left(\sum_{d,t}x_{a,d,t}+2y_{a,0}+5y_{a,1}-5\right), \tag{5}$$

where $y_{a,0}$ and $y_{a,1}$ take 0 or 1. By introducing these supplementary variables, we can handle multiple conditions in an expression. For example, in the case of $y_{a,0}=0$ and $y_{a,1}=0$, expression (5) results in the following expression:

$$\sum_a\left(\sum_{d,t} x_{a,d,t} - 4\right)\left(\sum_{d,t} x_{a,d,t} - 5\right),$$

and this takes minimum value 0 if the number of working days in a week ($\sum_{d,t} x_{a,d,t}$) is 4 or 5. Similarly, in the case of $y_{a,0}=1$ and $y_{a,1}=0$, it takes minimum value 0 if the number of working days in a week is 2 or 3. On the other hand, regardless of the combination of $y_{a,0}$ and $y_{a,1}$, if the number of working days is 1, 6, or 7, the value of equation (5) will be greater than 0.

Finally, the expression (1) is transformed into QUBO so that minimum value is taken when the number of workers who work zero days in a week becomes the maximum. The supplementary variable $y_{a,1}$ introduced in the transformation to expression (5) takes the maximum value 1 when the number of work days in a week for a worker a is 0. Therefore, the following values taking the total sum of $y_{a,1}$ and minus signs take the minimum value when the above conditions are satisfied:

$$-\sum_a y_{a,1}. \tag{6}$$

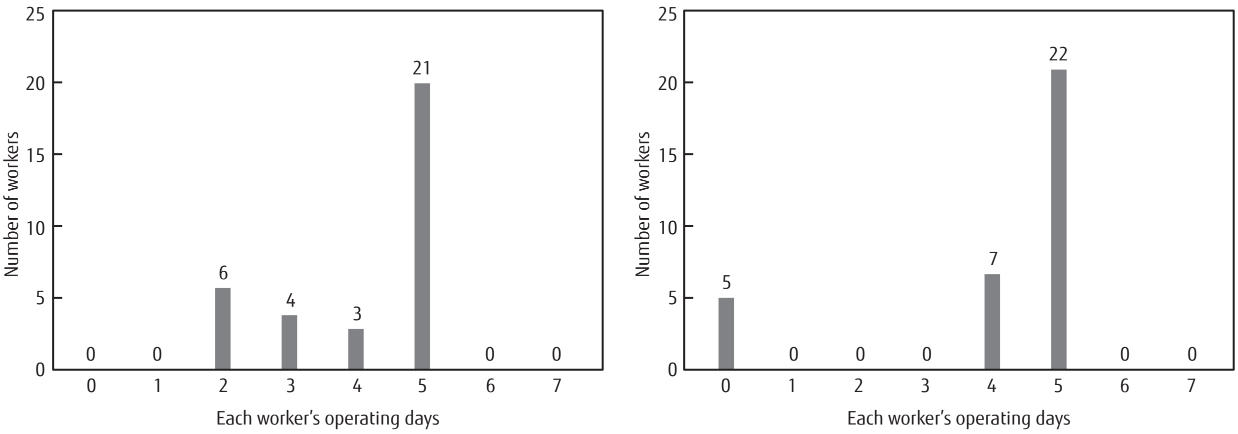
Through the above transformations, the optimization problem described by the objective function and multiple constraints can be described by the following single expression:

$$\min|(4)+(5)+(6)|.$$

To solve QUBO, it is necessary to create an equation considering that the value of each variable becomes 0 or 1. In this case, supplementary variables are used to express workers' working days and the range of minimization of assigned personnel. Fujitsu has several established patterns for solving QUBO, such as the use of supplementary variables that appeared in this case.

As a result, we found that worker assignment satisfying the constraint conditions can be automated. Furthermore, by assigning 4 or 5 days work to a part of workers, we were able to reduce the workers assigned to work. **Figure 5** (a) and (b) show the assignments of workers before and after the introduction of Digital Annealer. These figures show that, by introducing Digital Annealer, we have succeeded in allowing 29 workers to perform the work that was conducted by 34 workers before the introduction.

In this way, the conversion to QUBO (case of Step 4) can be realized by utilizing established techniques such as the use of supplementary variables.



(a) Assignment before the introduction of Digital Annealer

(b) Assignment using Digital Annealer

Figure 5
Comparison of workforce assignment using Digital Annealer.

7. Conclusion

In this paper, we explained mathematical problem formulation, which is a technology to solve customers' combinatorial optimization problems with Digital Annealer, and introduced use cases.

Fujitsu has been compiling the know-how regarding the aforementioned five steps by conducting proof of concept (PoC) with individual users (customers) before the launch of the product. We will continue making contributions to our customers' businesses by putting the know-how accumulated in this way to use in solving new problems in the future.

In the future, we will aim to expand the scope of problems to be handled. Our future challenges include adaptation to an on-site version, expansion of the effective scale and number of gradations of couplings, and enhanced functions to automate parameter adjustment.

Furthermore, we will aim to create a new global computing market. In order to realize this, we began aggressively promoting the development of human resources, developing business platforms, and pursuing co-creation with customers and partners in North America, an advanced market in terms of quantum computing. Using these efforts as a foothold, we aim to expand the Digital Annealer business in the global market.

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