Global Trends and Our Efforts to Realize Early-FTQC

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QIQB

RIKEN QUANTUM COMPUTING

Outline

- Motivation for early fault-tolerant quantum computing
- Partially fault-tolerant quantum computing

  “***Partially Fault-tolerant Quantum Computing Architecture*** *with Error-corrected Clifford Gates and Space-time Efficient Analog Rotations*”
  Y Akahoshi, K Maruyama, H Oshima, S Sato, KF, PRX Quantum (2024).
- Resource estimate for QPE with improved analog rotation gate

  "*Practical quantum advantage on partially fault-tolerant quantum computer.*" Toshio *et al.*
  arXiv:2408.14848.
- Reducing overhead of magic state distillation for earlyFTQC

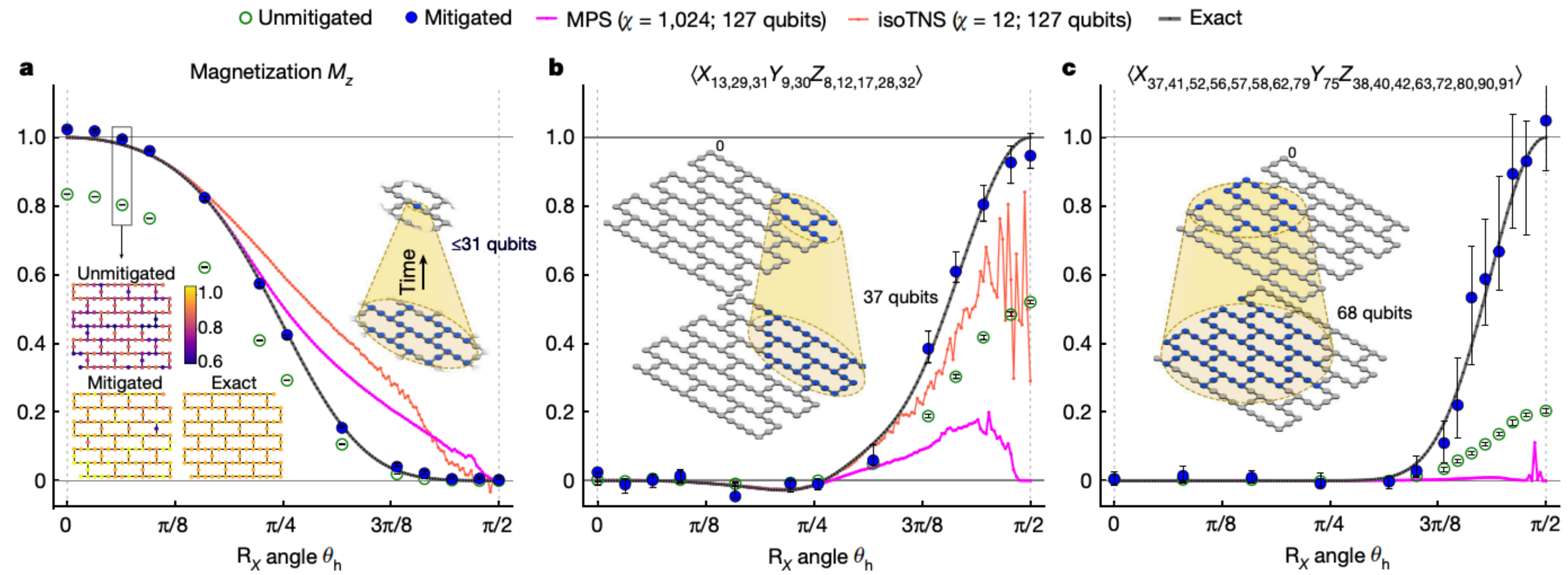
  "*Even more efficient magic state distillation by zero-level distillation.*"
  T. Itogawa et al. arXiv:2403.03991.
- Summary

Experimental Quantum Computing with >100-qubit scale system

Simulation of kicked Ising model with 127 qubits by IBM

Y. Kim et al. *"Evidence for the utility of quantum computing before fault tolerance."* Nature **618,** 500 (2023).

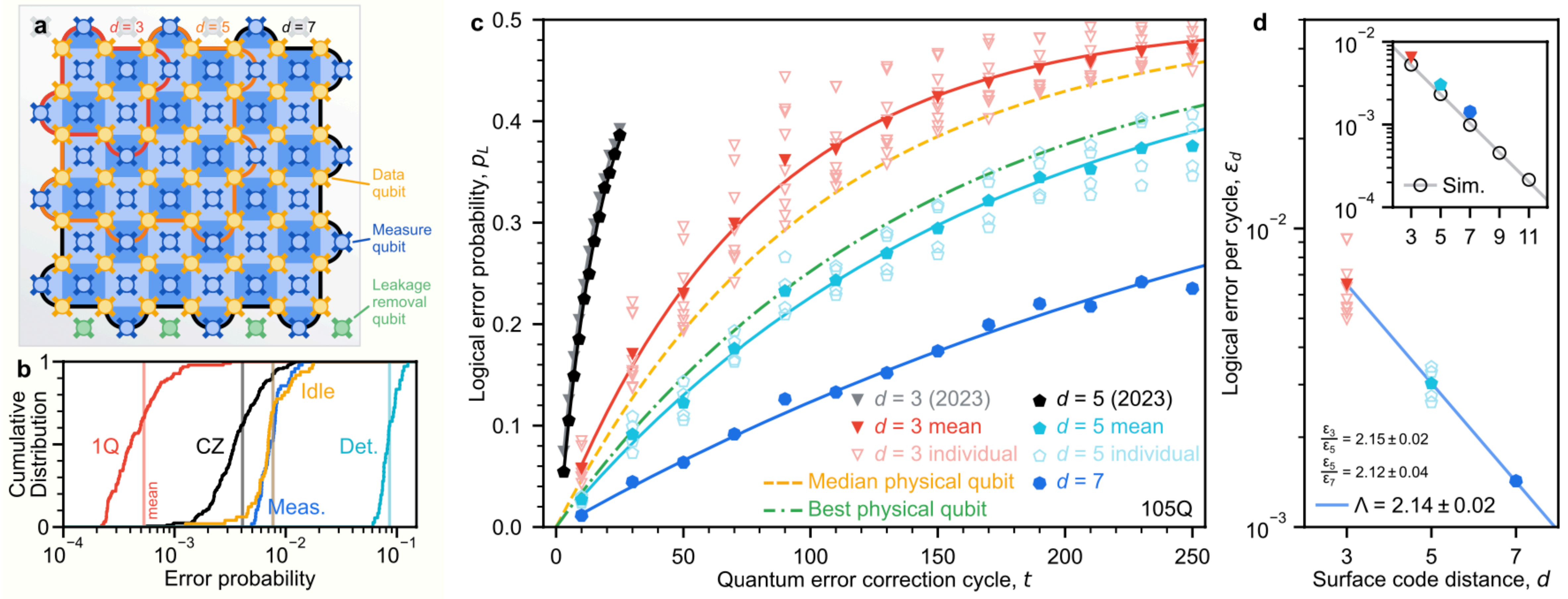
- 60 layers of CNOT gates = 2880 CNOTs
- Successful quantum error mitigation



Quantum error correction below surface code threshold

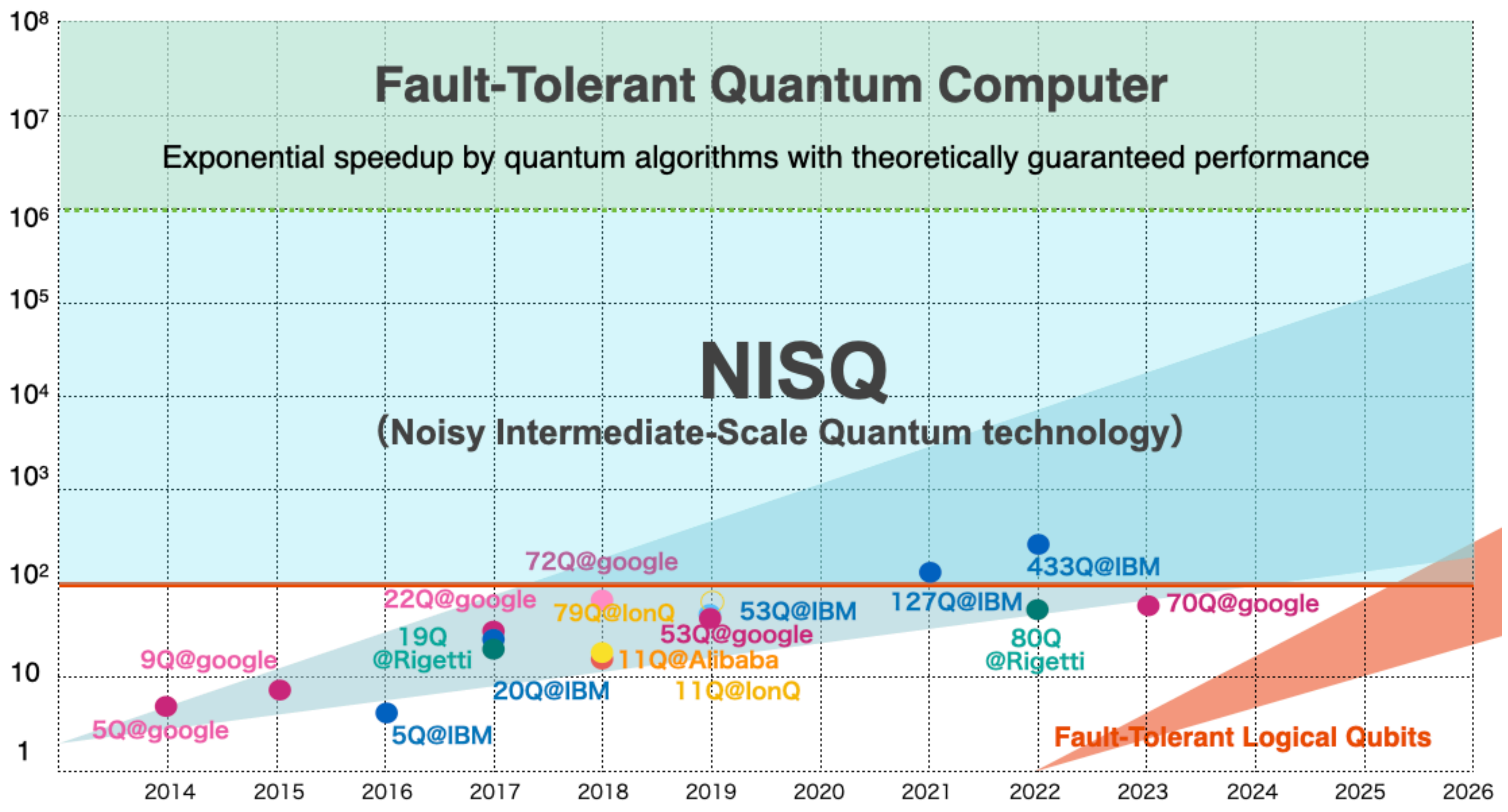
Google Quantum AI et al *"Quantum error correction below the surface code threshold*", Nature **638**, 920 (2024)

- 101-qubit, distance 7 surface code
- Error suppression factor Λ>2
- distance 5 realtime decoding (63μsec latency)
- logi. life time 291μsec > phys. one 85μsec



How can we close the gap between NISQ and FTQC?

- State-of-the-art quantum computer, ~100qubits, can perform tasks that are intractable for classical computer (e.g. *random quantum circuit sampling*).



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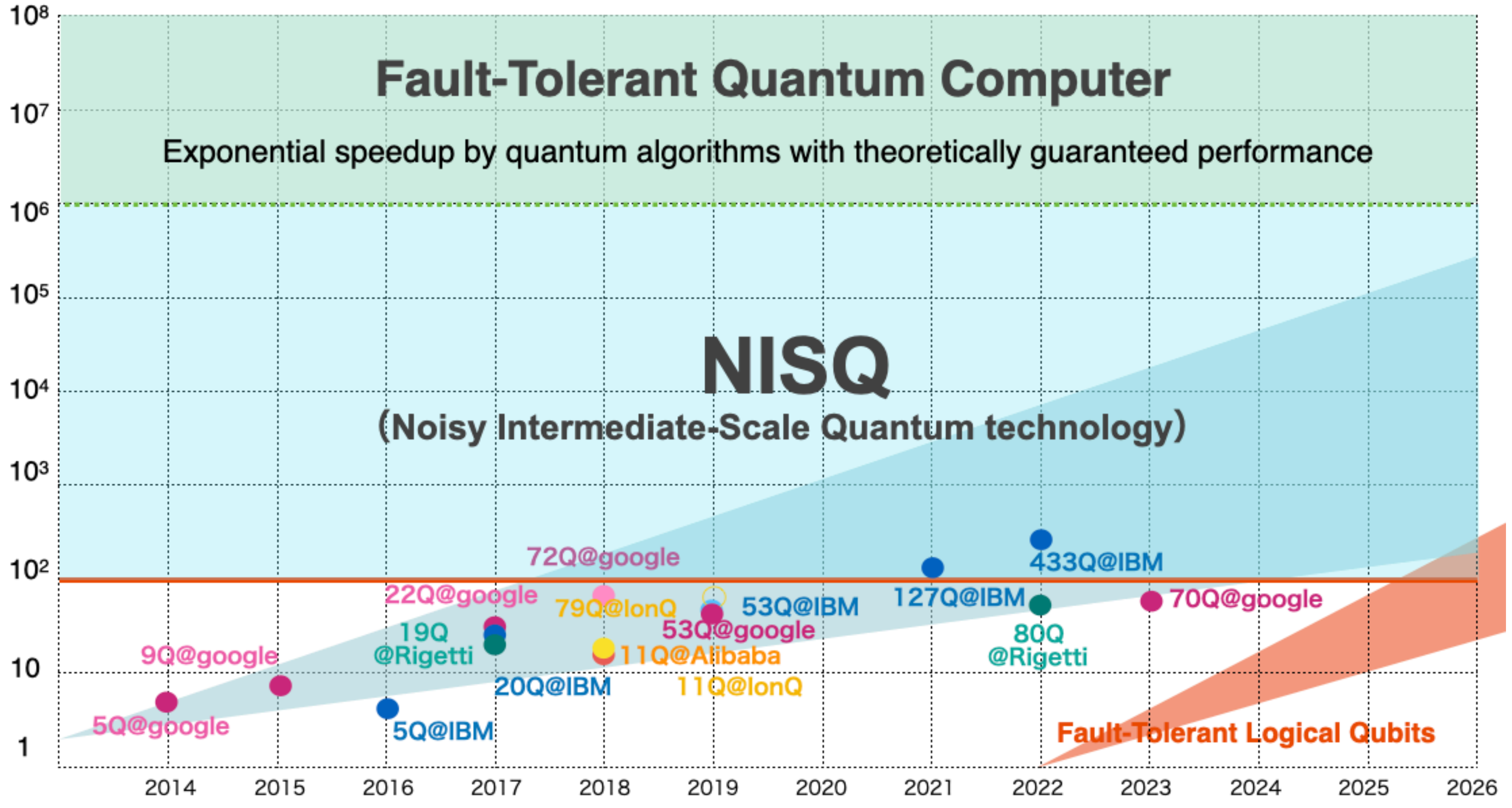
- For provable quantum advantage we need a fully fledged fault-tolerant quantum computer, requiring 1M qubits.



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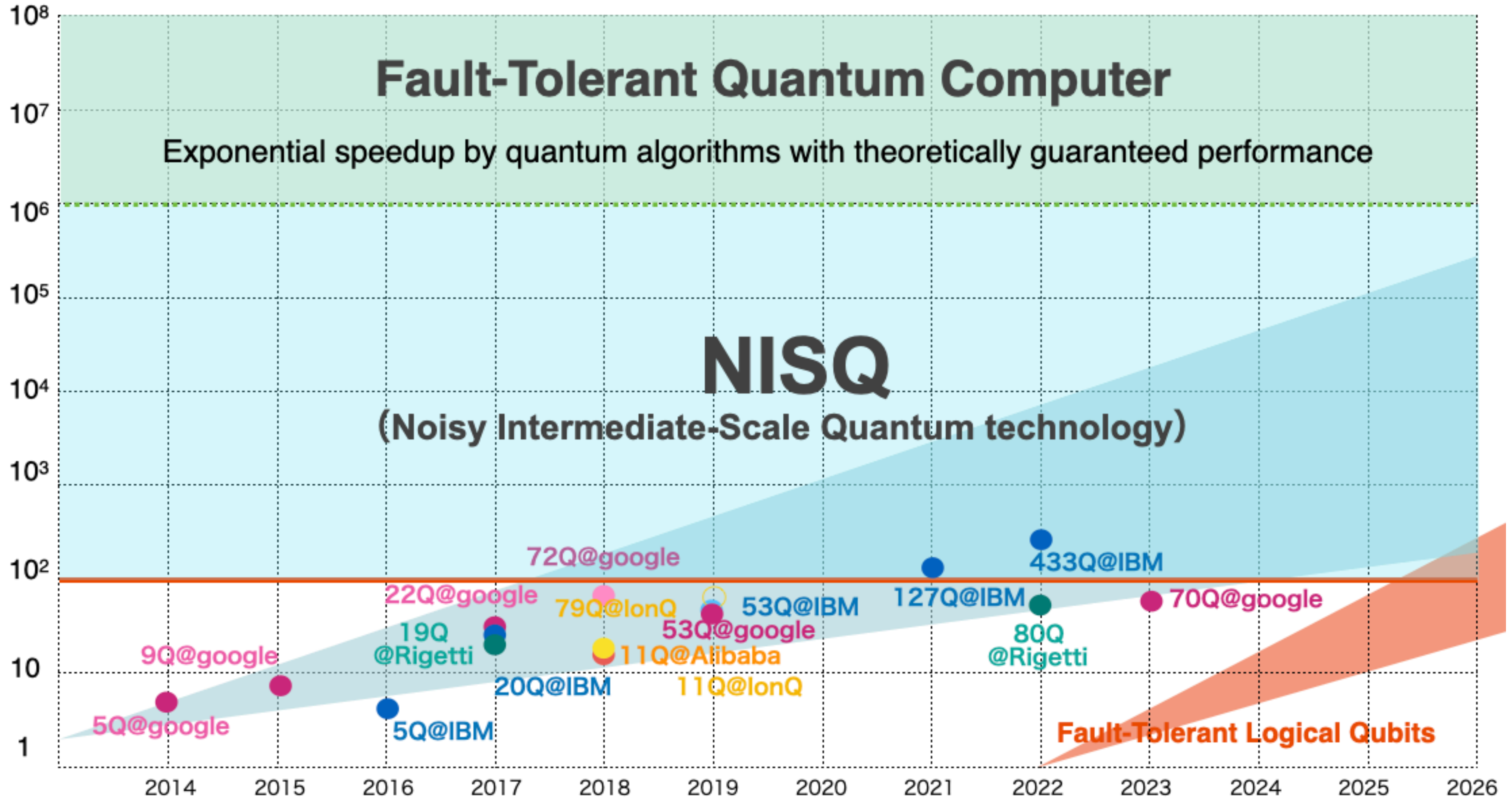


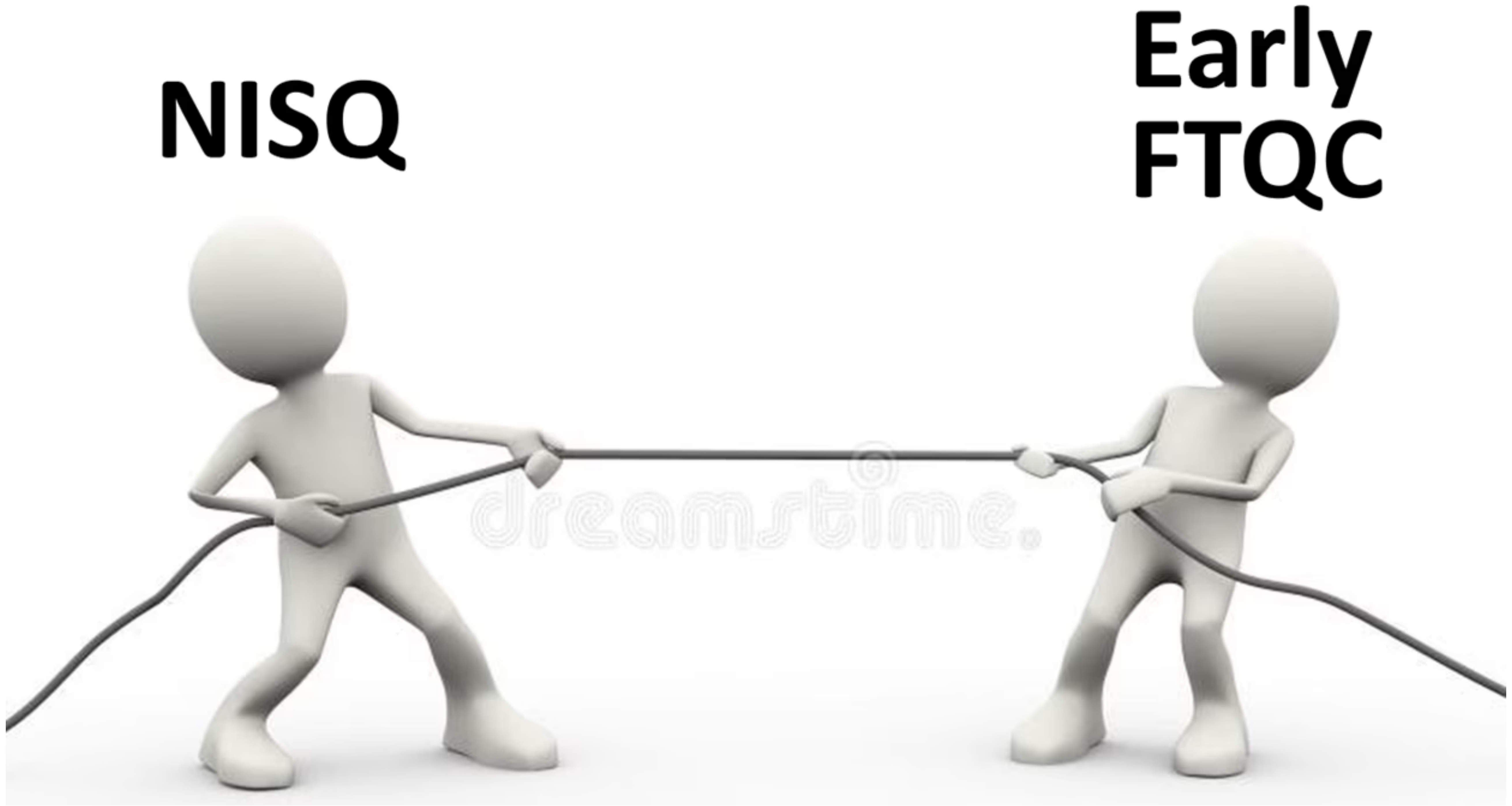
Putting a milestone between NISQ and FTQC is important for sustainable development of QC.

Early Fault-Tolerant Quantum Computing

- **NISQ**: >100 qubits, below 10^{-3} error rate, utility w/o error correction.

- **QEC break even**: >100 physical qubits, logical error rate $<10^{-3}$

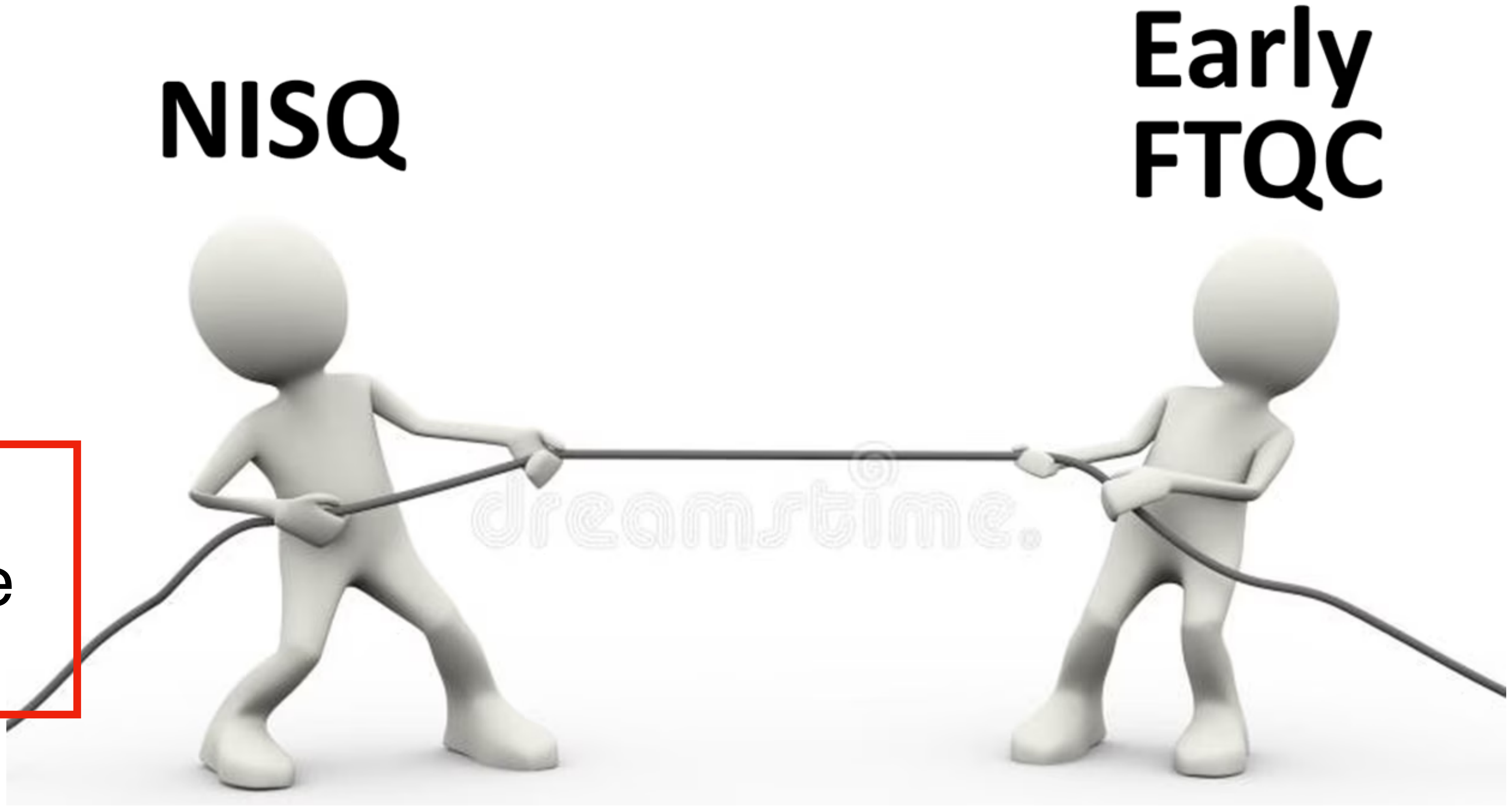
- **FTQC**: >1M physical qubits (>100 logical qubits), logical error rate $< 10^{-10}$, fully scalable



https://quantumcomputingreport-com.cdn.ampproject.org/c/s/quantumcomputingreport.com/nisq-versus-ftqc-in-the-2025-2029-timeframe/amp/

Early Fault-Tolerant Quantum Computing

- **NISQ**: >100 qubits, below 10^{-3} error rate, utility w/o error correction.
- **QEC break even**: >100 physical qubits, logical error rate <10^{-3}
- **early FTQC:** 1k-10k physical qubits (tens-100 logical qubits), logical error rate 10^{-4}-10^{-6}
- **FTQC**: >1M physical qubits (>100 logical qubits), logical error rate < 10^{-10}, fully scalable



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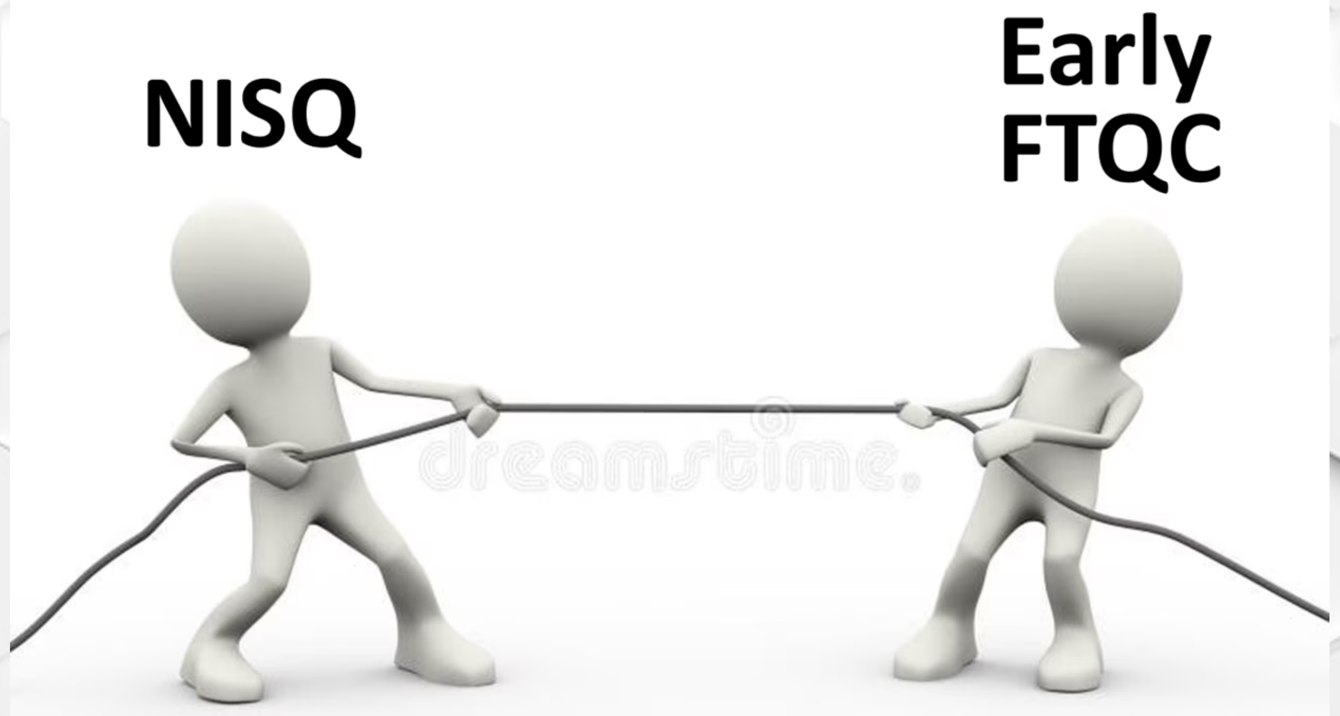
Partially fault-tolerant quantum computing

"*Partially Fault-tolerant Quantum Computing Architecture with Error-corrected Clifford Gates and Space-time Efficient Analog Rotations*"
Y Akahoshi, K Maruyama, H Oshima, S Sato, KF, PRX Quantum (2024).



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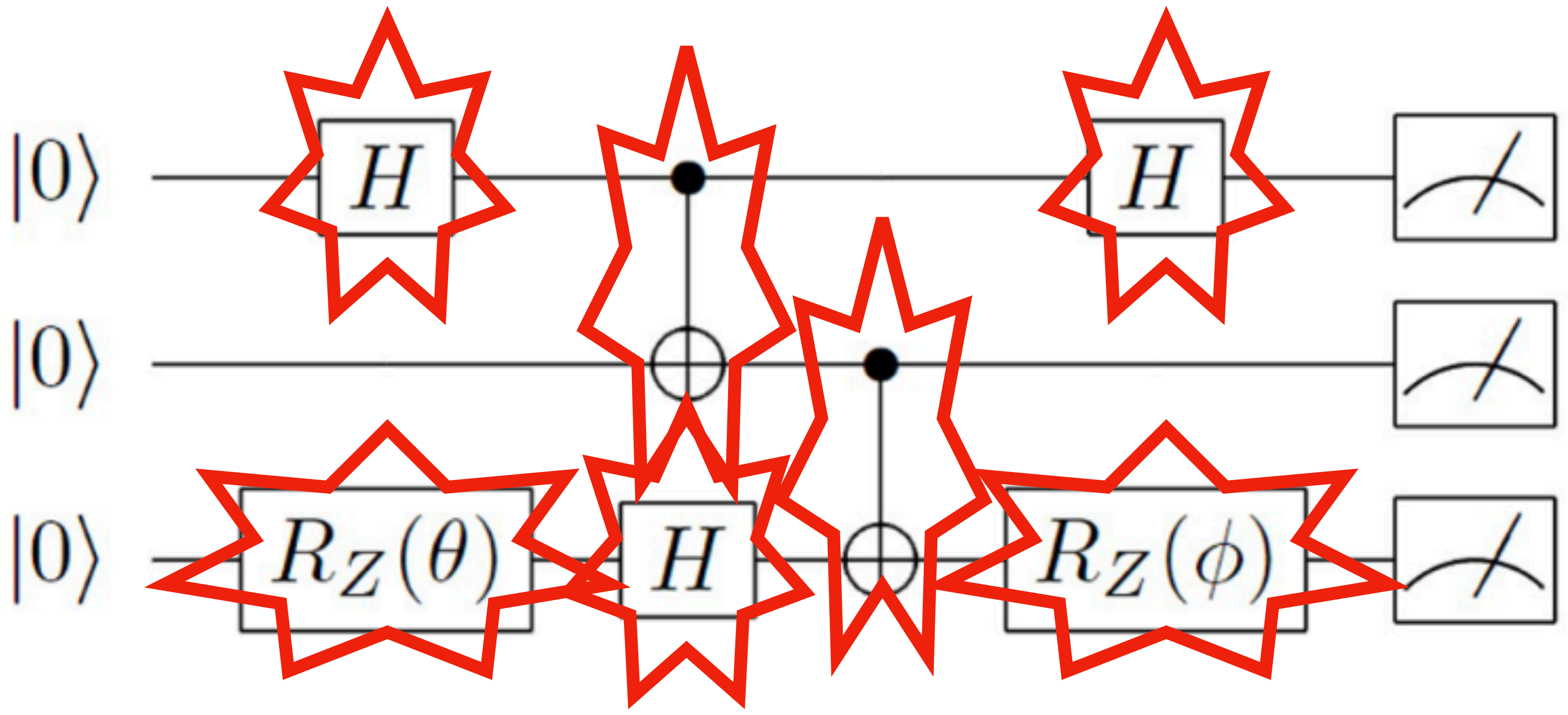
Our approach

NISQ

All operations are subject to errors.



Operations are limited by
physical qubit connectivity

$F = (1 - p)^{\text{Cliff + non-Cliff}}$

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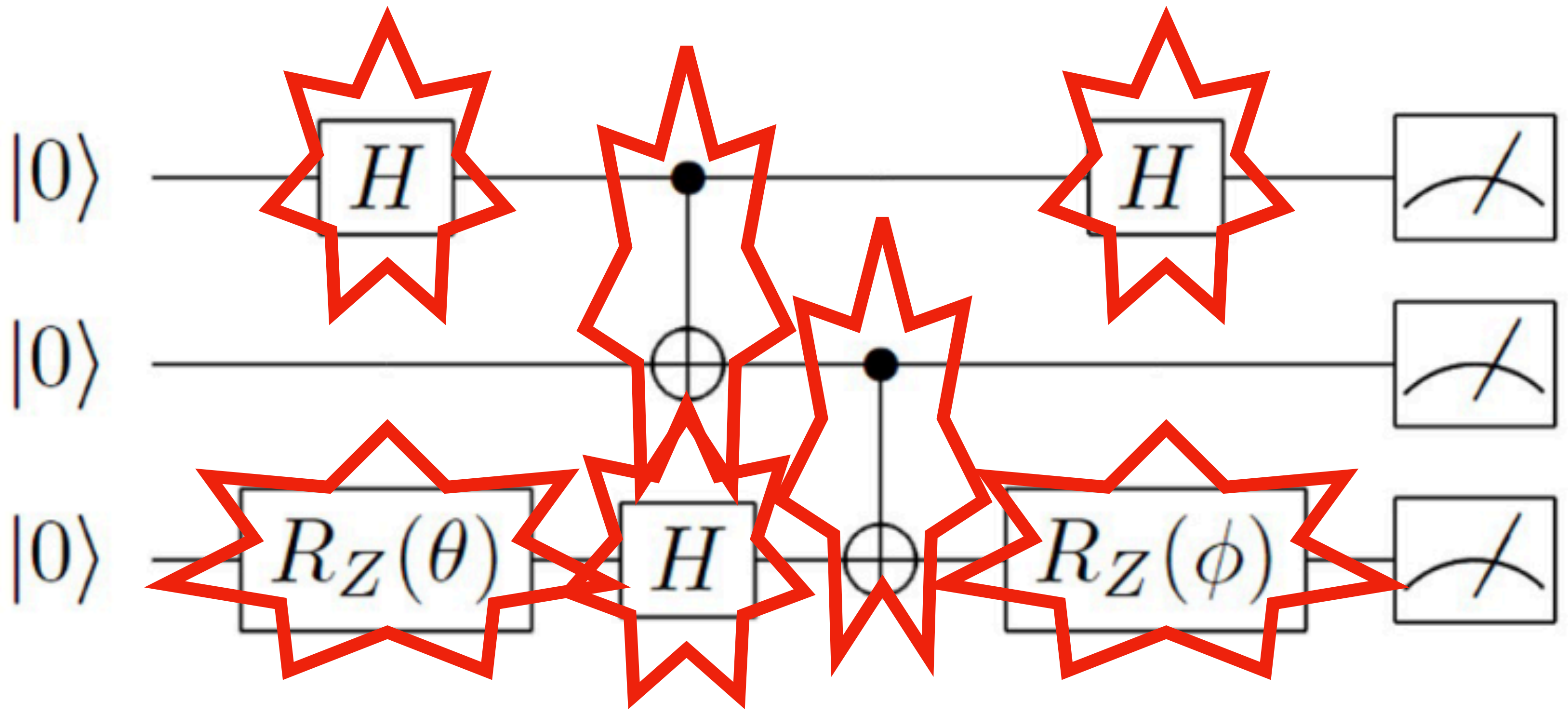

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FTQC

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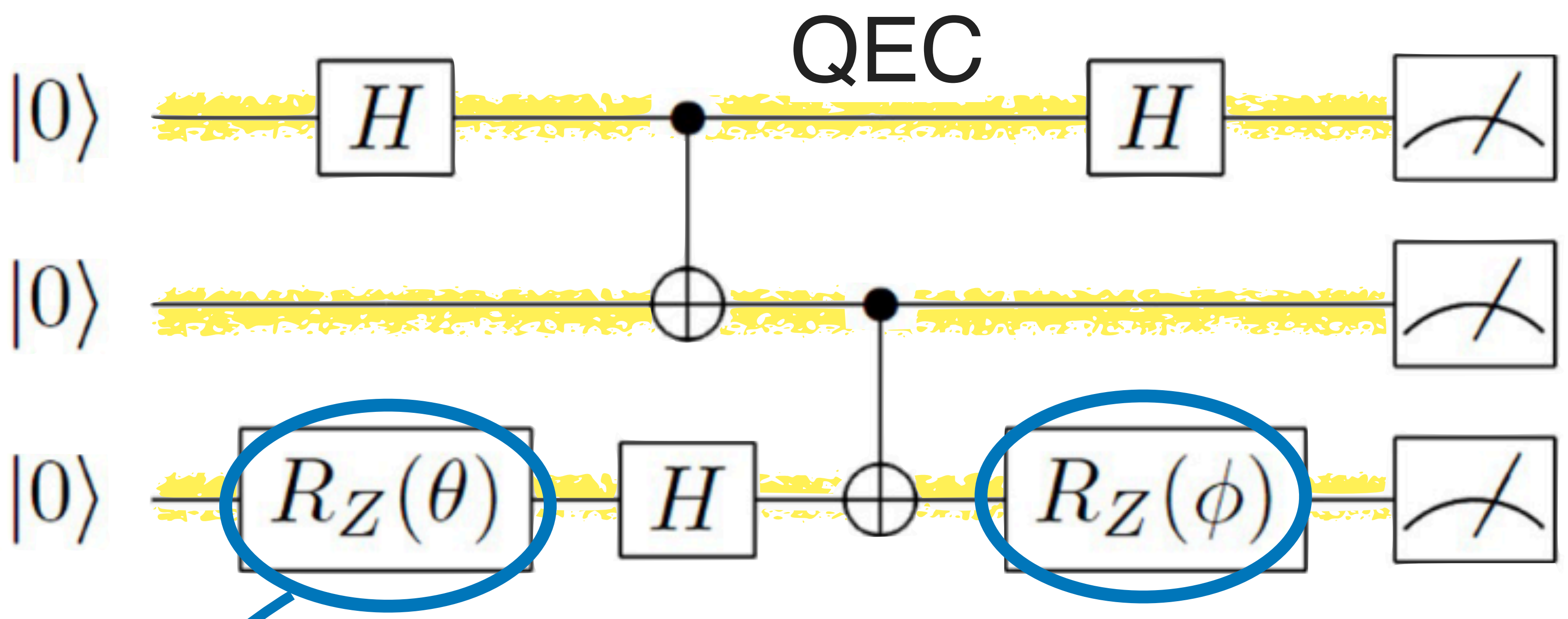

large space-time overhead
for magic state

$F = (1-p_L)^{\text{Cliff +non-Cliff}}$

$U = $ HTSHTSHTSHTHTHTHTSHTHTSHTSHTSHTHTHTSHTSHTHTHTSHTHTSHTHTHTHTHTHTHTSHTSHT
SHTHTSHTHTSHTHTHTHTSHTHTHTSHTHTSHTHTHTHTSHTSHTSHTHTHTSHTSHTSHTSHTHTSHTS
HTSHTSHTHTSHTHTSHTSHTHTHTHTHTSHTHTHTHTSHTSHTSHTHTSHTSHTHTHTSHTHTHTHTHTS
HTSHTHTHTHTHTSHTHTHTHTSHTHTHTHTHTHTHω^7

Our approach

NISQ

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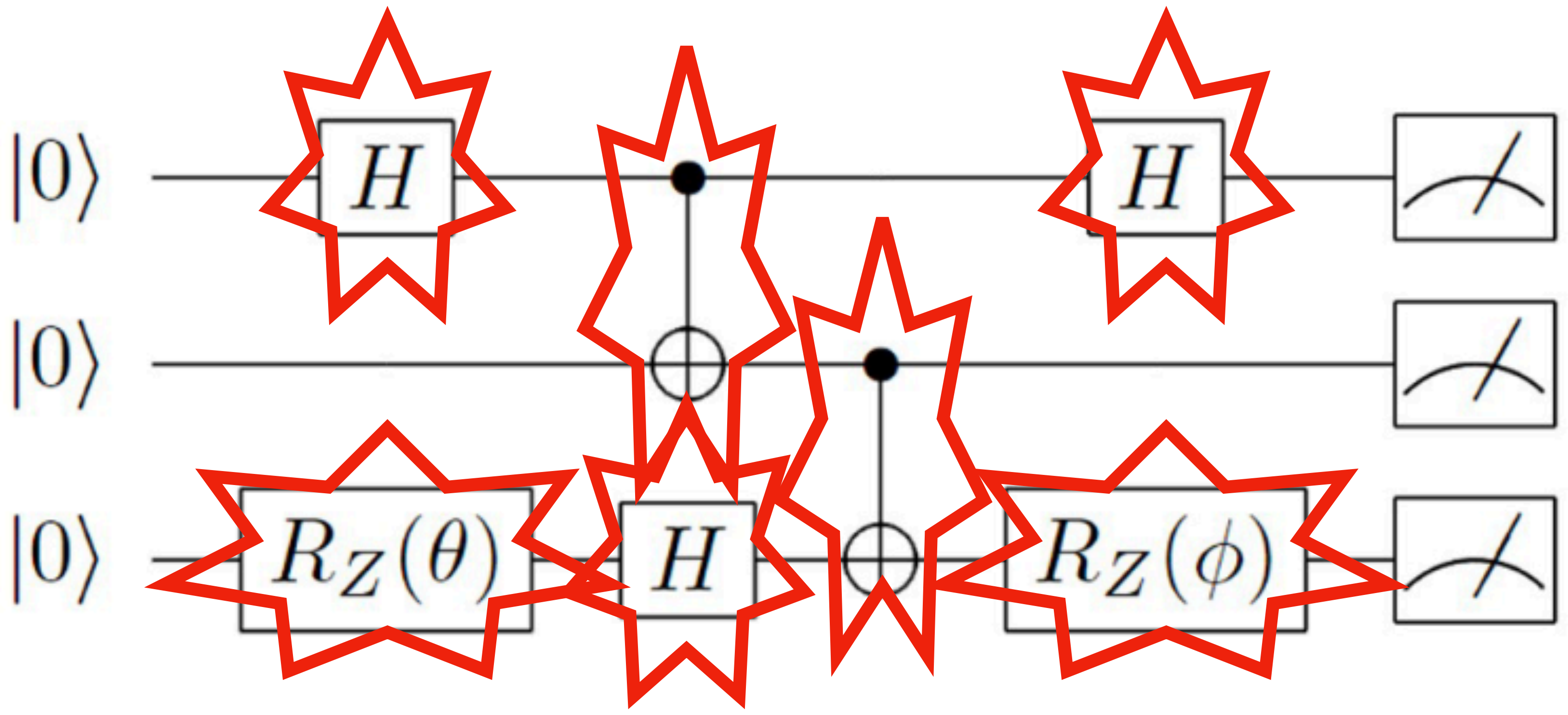

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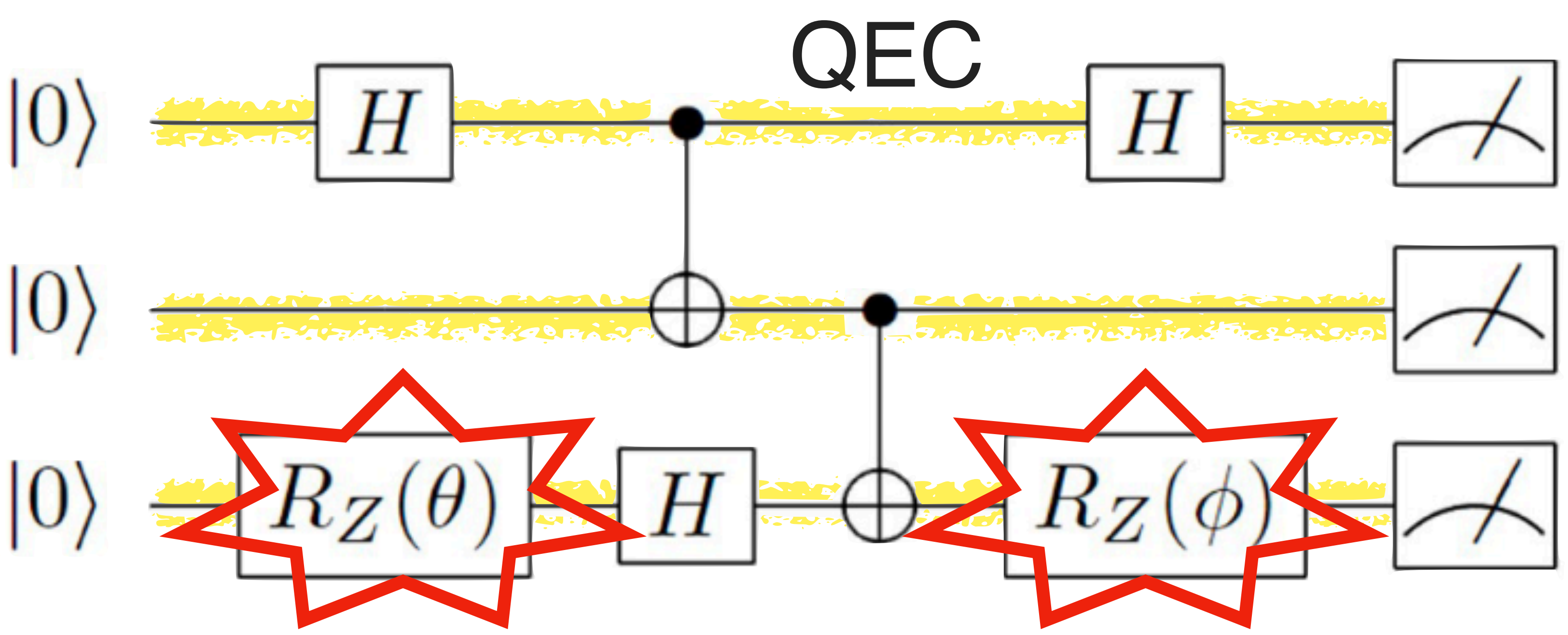
earlyFTQC



All-to-all connections are available at the logical level.

$F = (1 - p_L)^{\text{Cliff}}(1 - p)^{\text{non-Cliff}}$

FTQC

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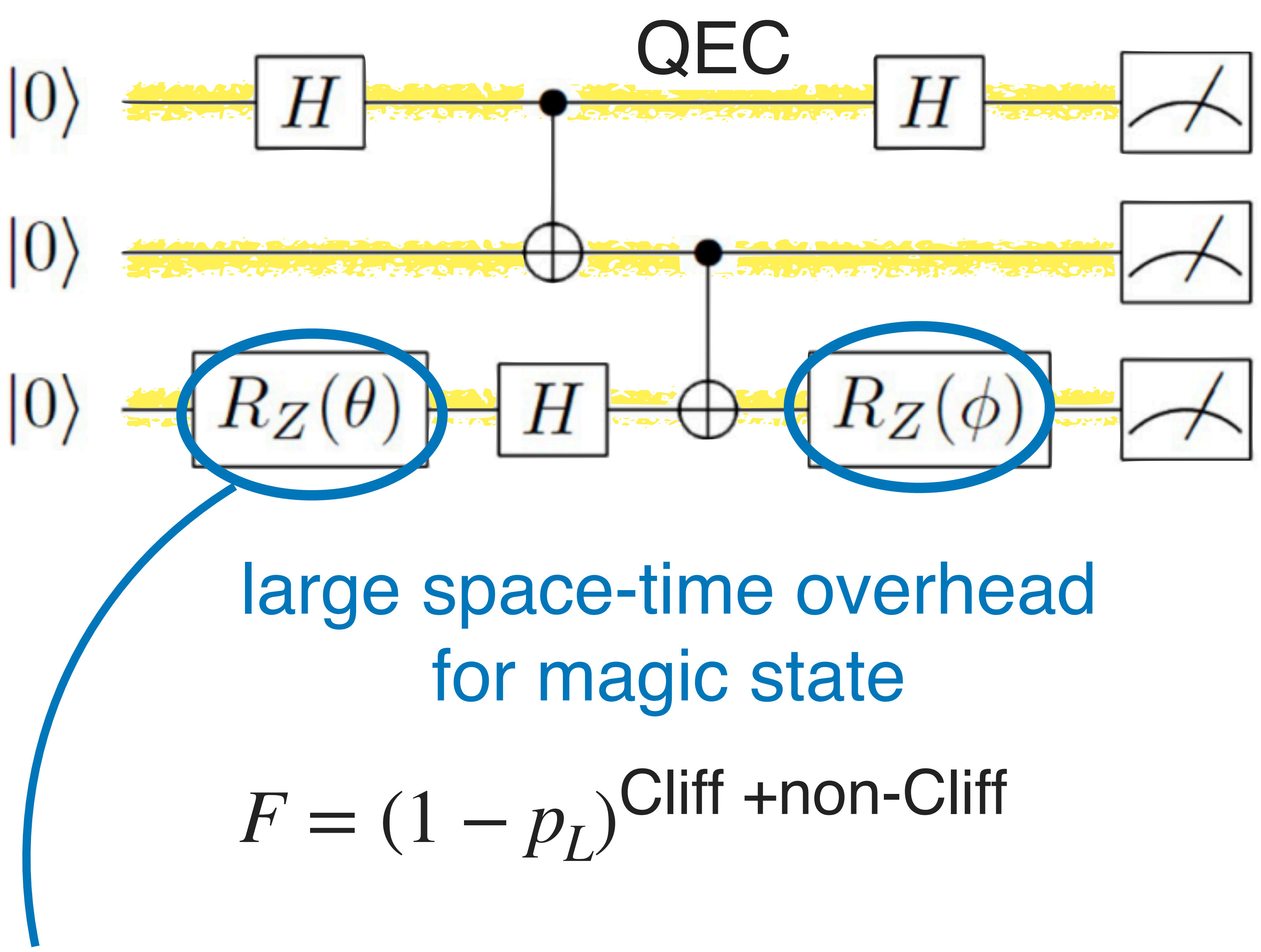

large space-time overhead for magic state

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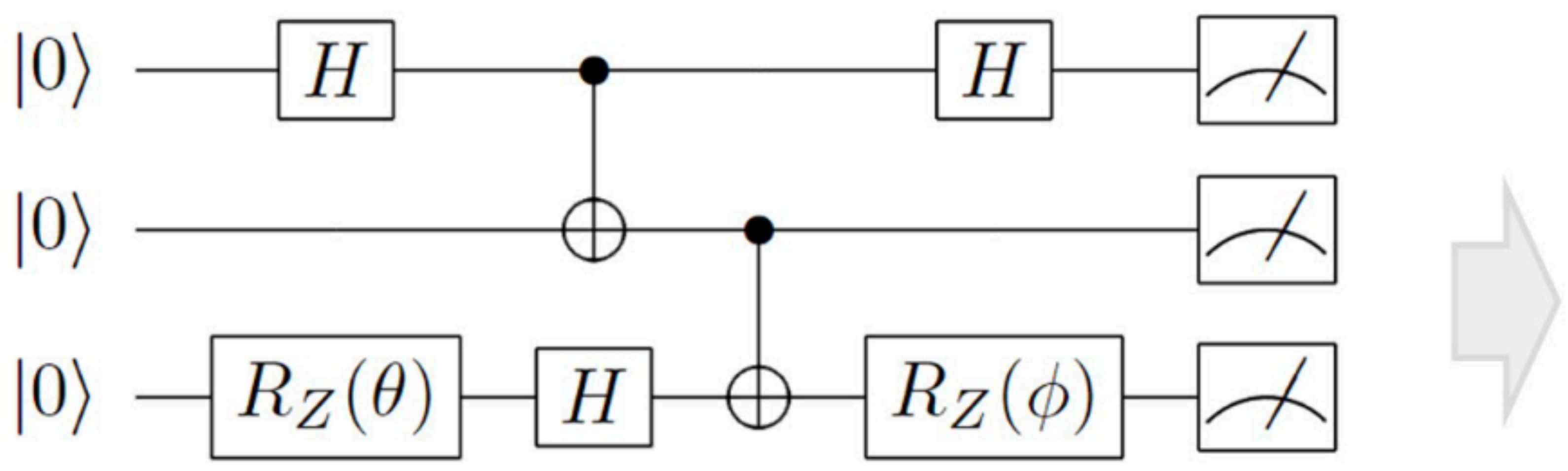
Partially Fault-tolerant Quantum Computing Architecture

Y Akahoshi, K Maruyama, H Oshima, S Sato, KF, PRX Quantum **5**, 010337 (2024).

Assumption: 10,000 qubits with 10^{-3}-10^{-4} error rate.



Quantum circuit

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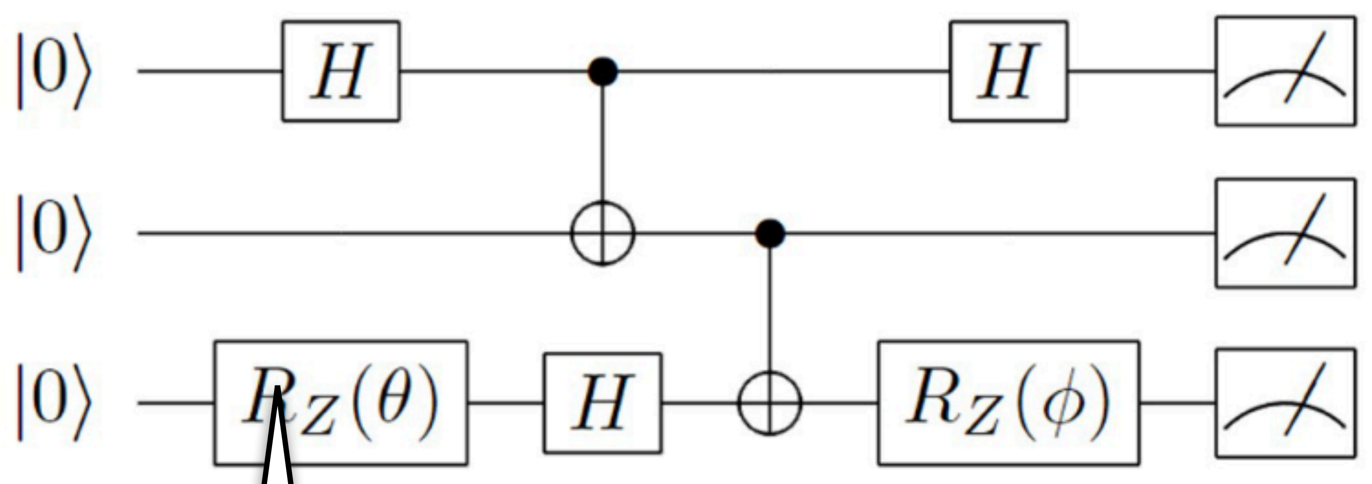

Quantum circuit

Clifford + T decomposition for $R_z(\pi/128)$:

$U =$ HTSHTSHTSHTHTHTHTSHTHTSHTSHTSHTHTHTSHTSHTHTHTSHTHTSHTHTHTHTHTHTHTSHTSHT
SHTHTSHTHTSHTHTHTHTSHTHTHTSHTHTSHTHTHTHTSHTSHTSHTHTHTSHTSHTSHTSHTHTSHTS
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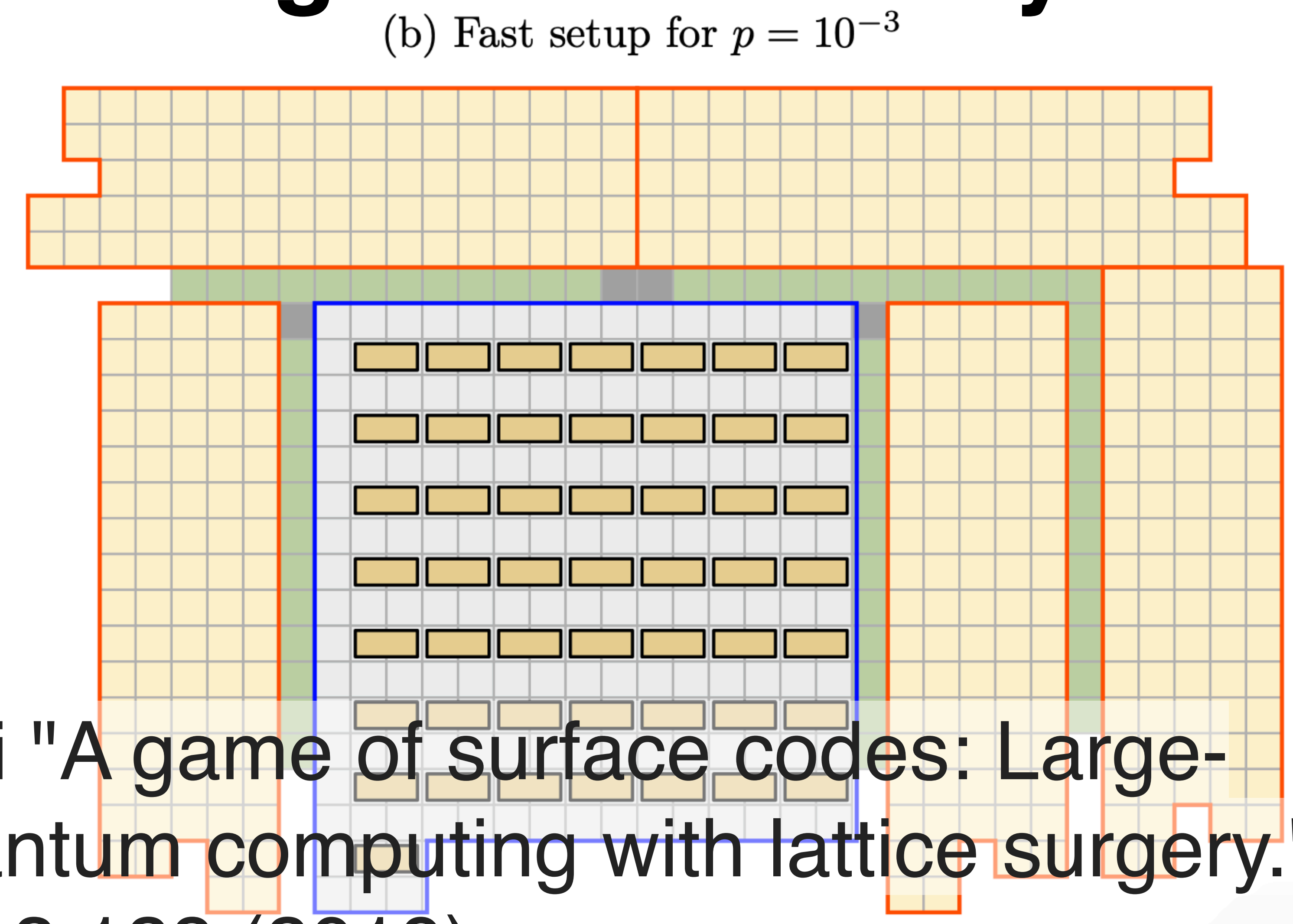
N. J. Ross and P. Selinger.
"Optimal ancilla-free Clifford+ T approximation of z-rotations." arXiv:1403.2975.

magic state factory



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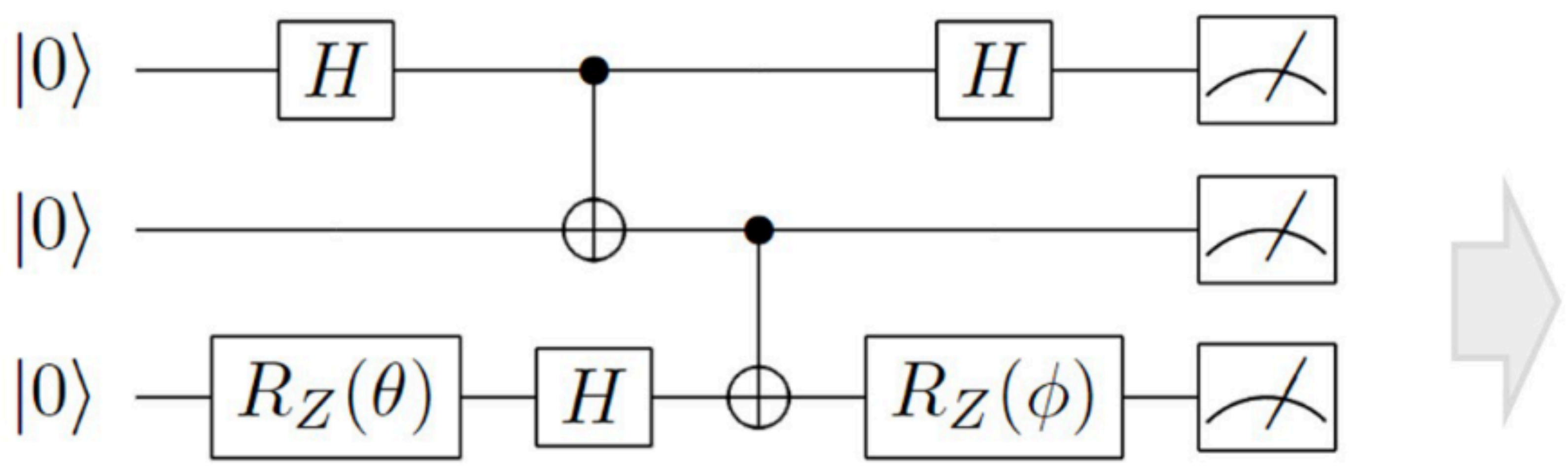

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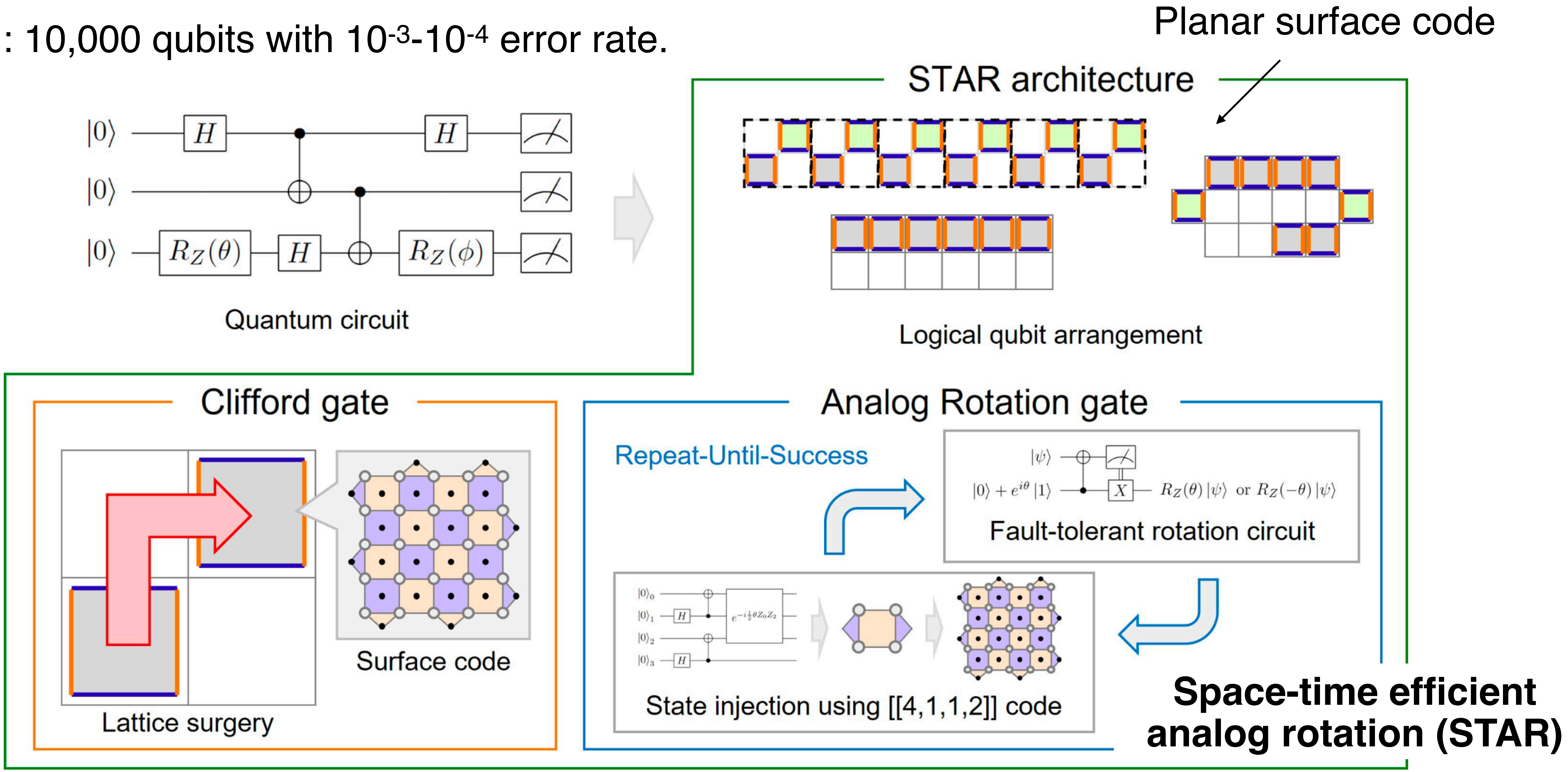


Clifford gates are fully protected.

1Q rotations are not protected but fairly accurate.

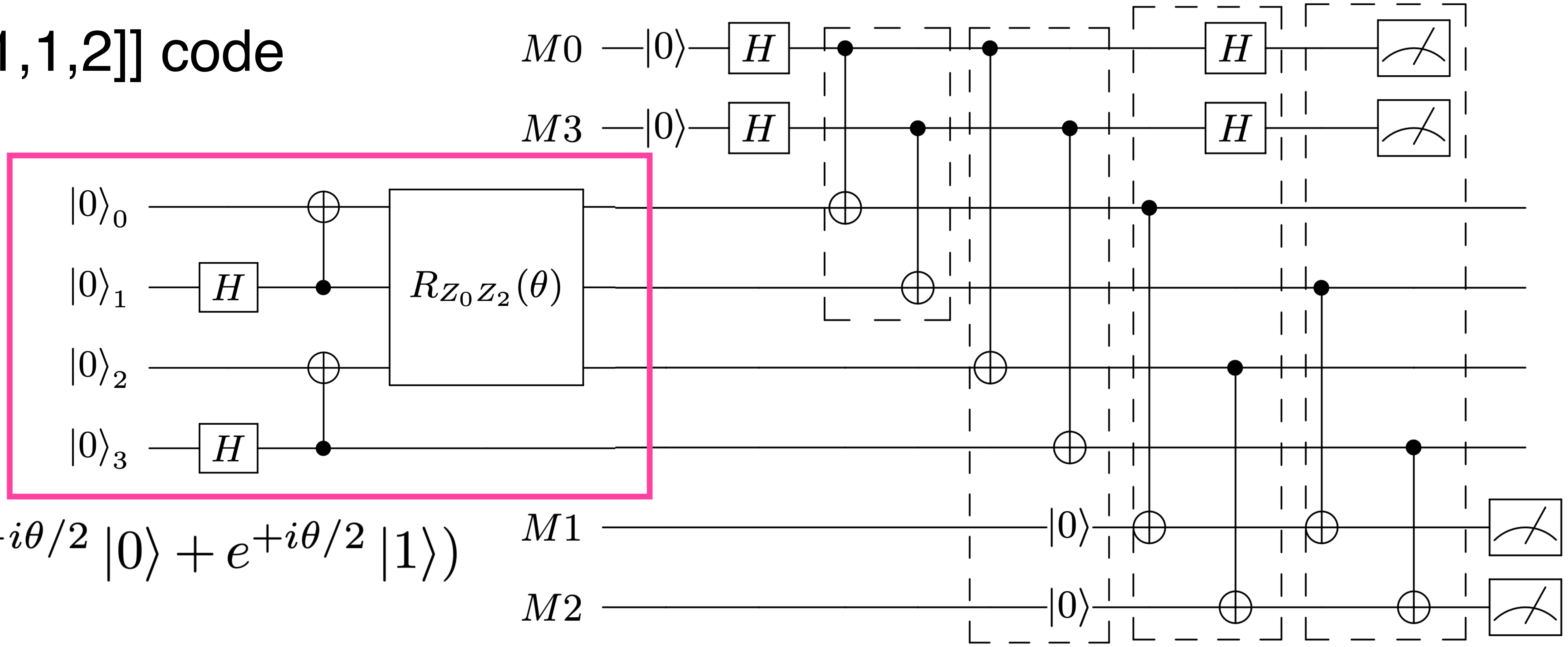
Quantum advantage is achievable with 10,000qubits and 10^{-4} error rate.

Y Akahoshi, K Maruyama, H Oshima, S Sato, KF, PRX Quantum **5**, 010337 (2024).

Ancilla injection with error detection

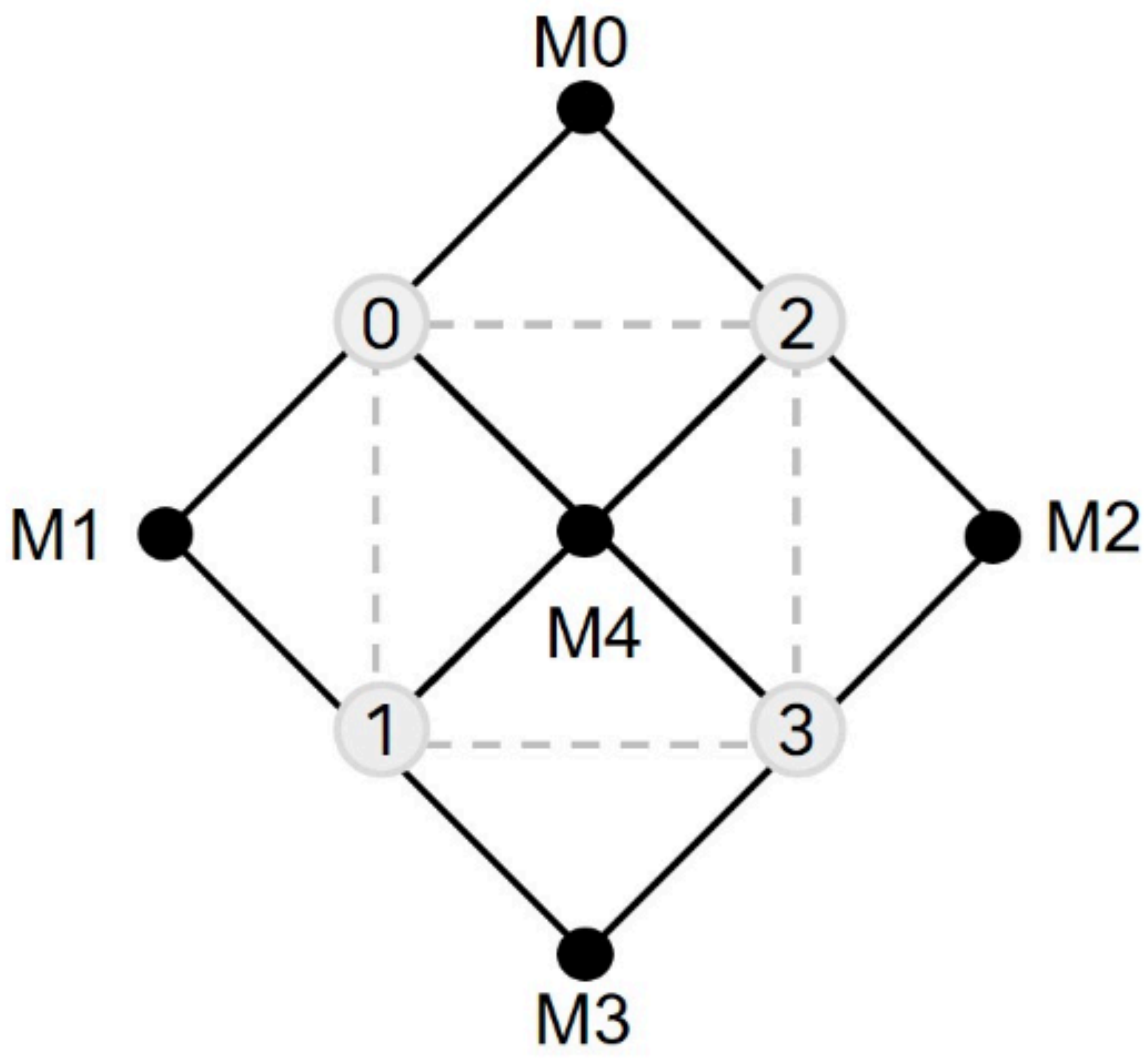
Ancilla state generation for analog rotation gate

[[4,1,1,2]] code



$$\frac{1}{\sqrt{2}}(e^{-i\theta/2}|0\rangle + e^{+i\theta/2}|1\rangle)$$

Errors are detected and discarded
except for unavoidable one



Stabilizer: XXXX, ZZZZ

Logical op: Z_0Z_2, X_0X_1

Gauge op: X_0X_2, Z_0Z_1

Resource estimate for QPE with improved analog rotation gate

Toshio *et al.* "*Practical quantum advantage on partially fault-tolerant quantum computer.*" arXiv:2408.14848.

List of quantum algorithms possibly done on earlyFTQC

- Simulation of quantum many-body dynamics **beyond physical qubit connectivities**

- Variational Quantum Algorithms **beyond physical qubit connectivities**

- Sampling-based eigensolver such as **Quantum-Selected Configuration Interaction (QSCI)**

  Kanno et al. *"Quantum-selected configuration interaction: Classical diagonalization of Hamiltonians in subspaces selected by quantum computers."* arXiv:2302.11320 (2023).

  J. Robledo-Moreno, et al. "Chemistry beyond exact solutions on a quantum-centric supercomputer." arXiv:2405.05068 (2024).

- **Quantum Phase Estimation**

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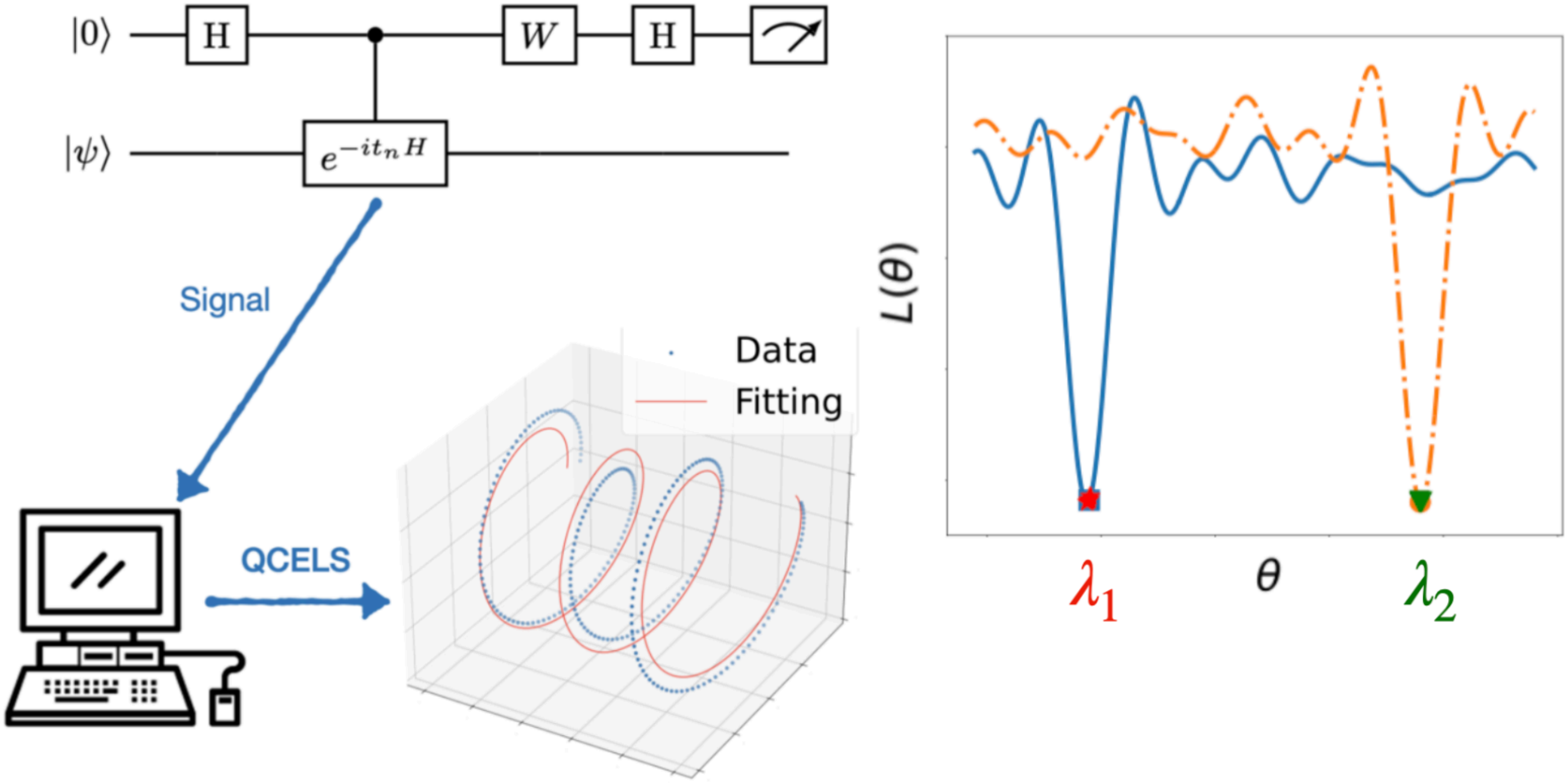
- **Quantum Phase Estimation**

EarlyFTQC version of Quantum Phase Estimation

Quantum complex exponential least squares (QCELS)

Ding et al, "Even shorter quantum circuit for phase estimation on early fault-tolerant quantum computers with applications to ground-state energy estimation." PRX Quantum (2024); see also Quantum (2023).



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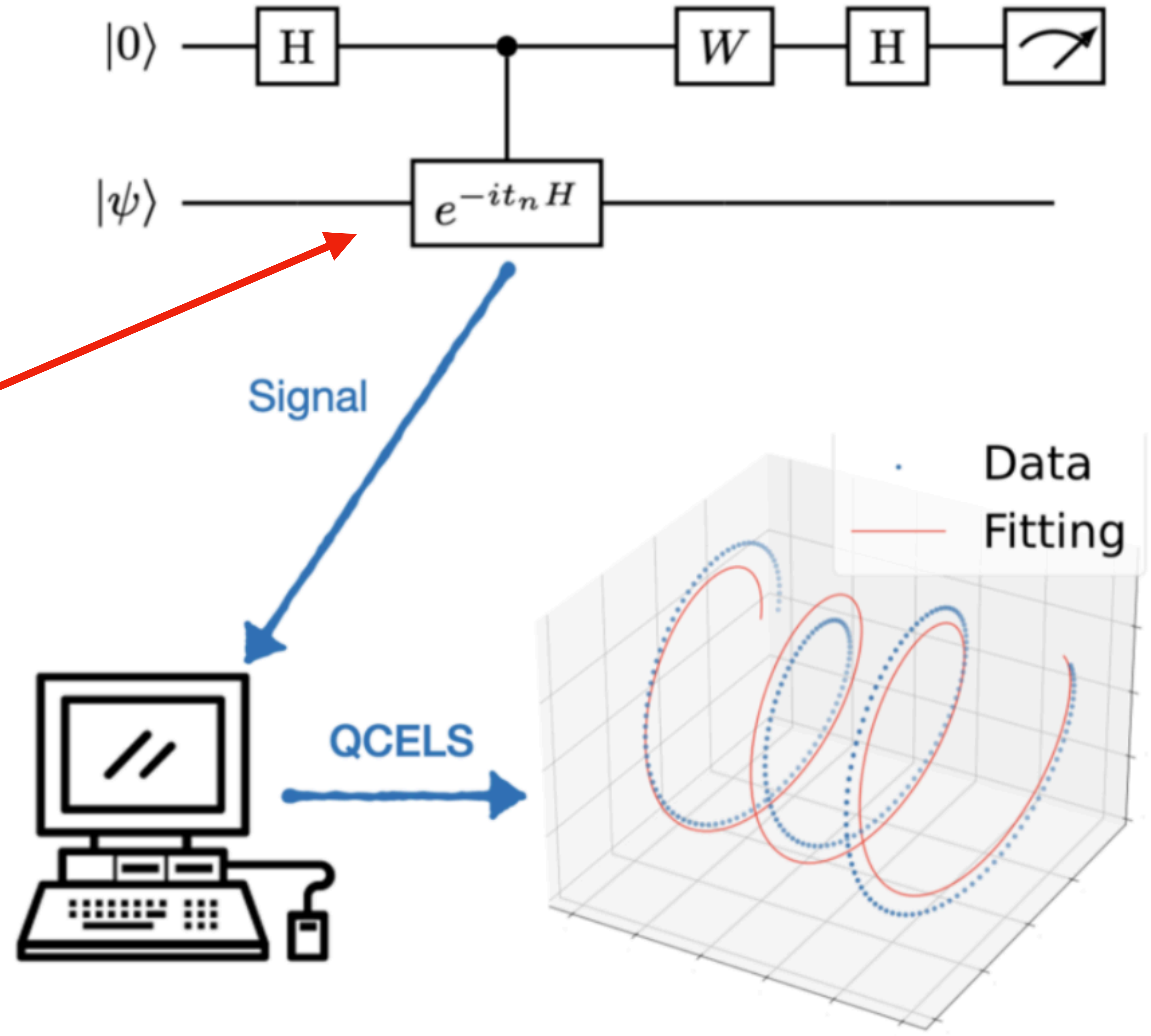
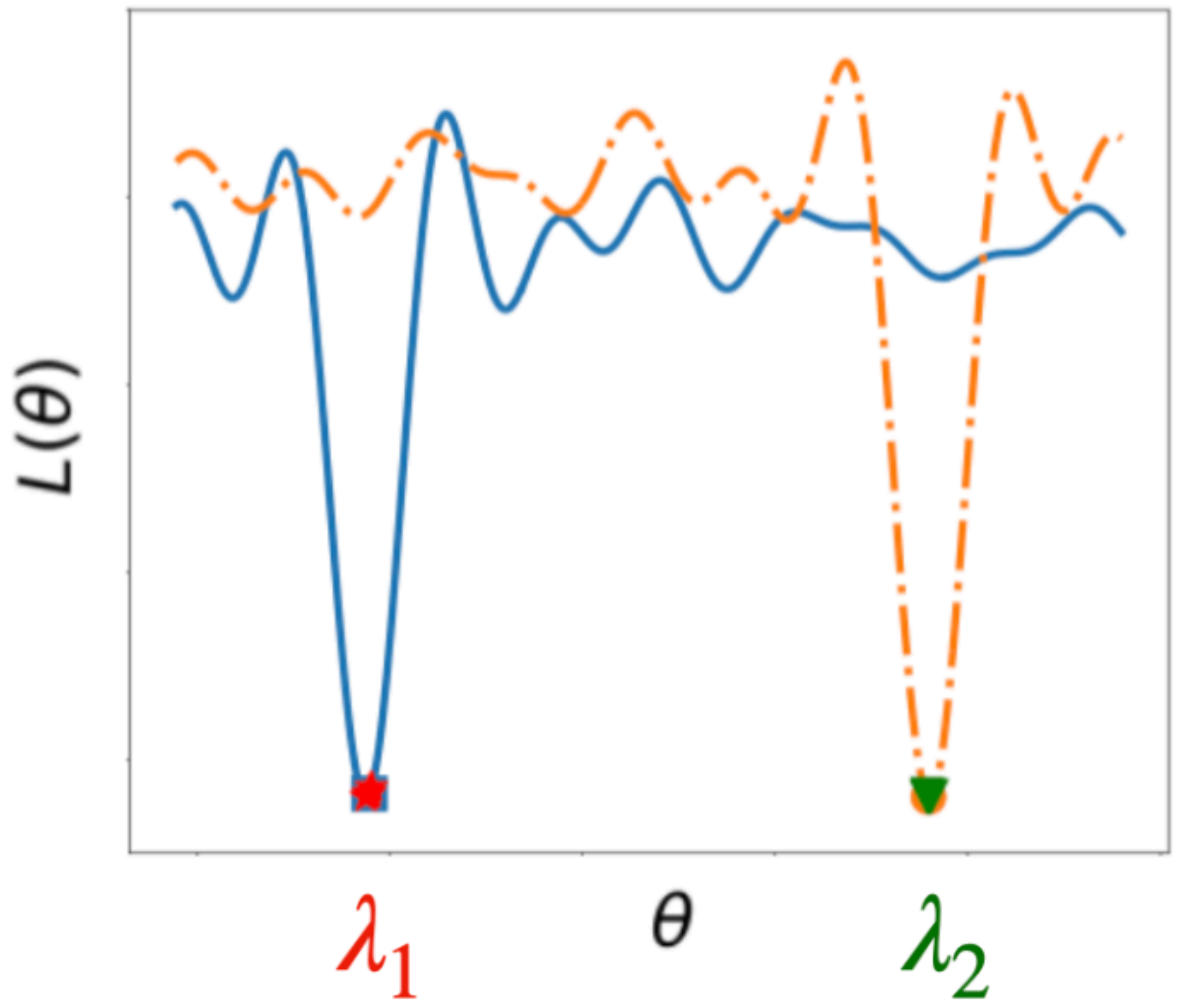




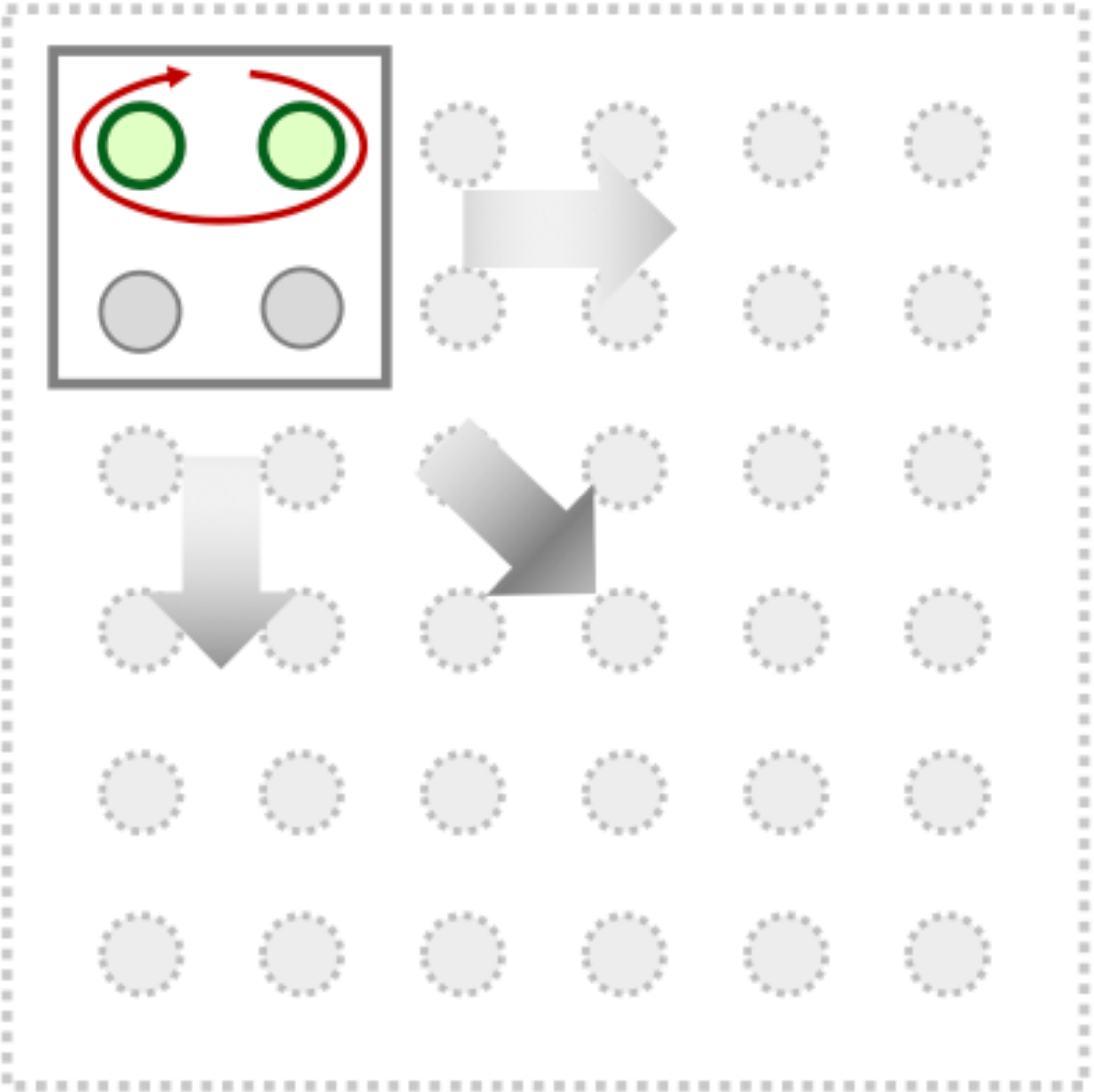
Trotterization

$$e^{-iHt} = \left[\prod_k e^{-i(t/N)P_k}\right]^N$$

a lot of
small angle rotations

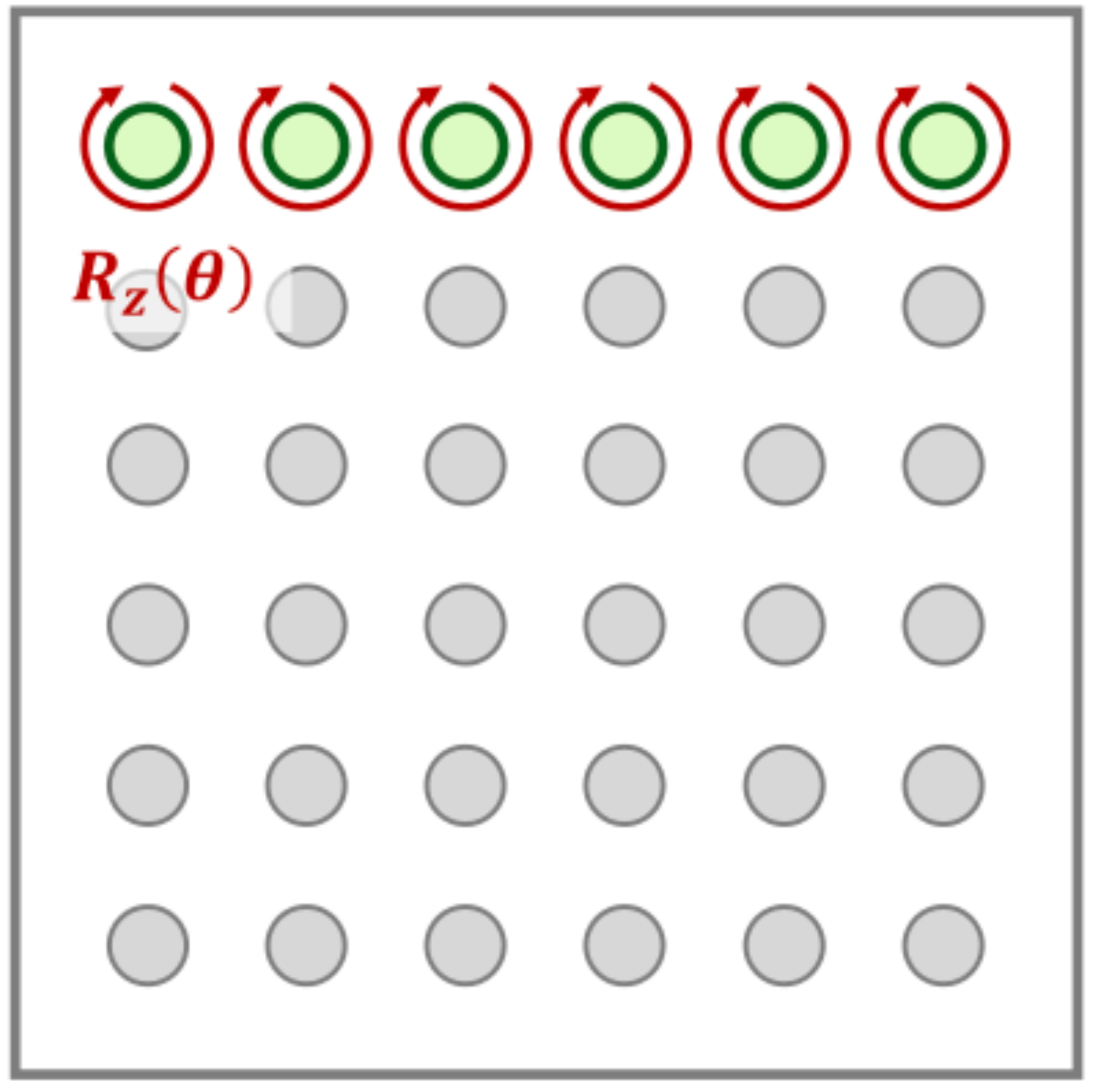
Error reduction for small angle rotation gate

Our previous approach



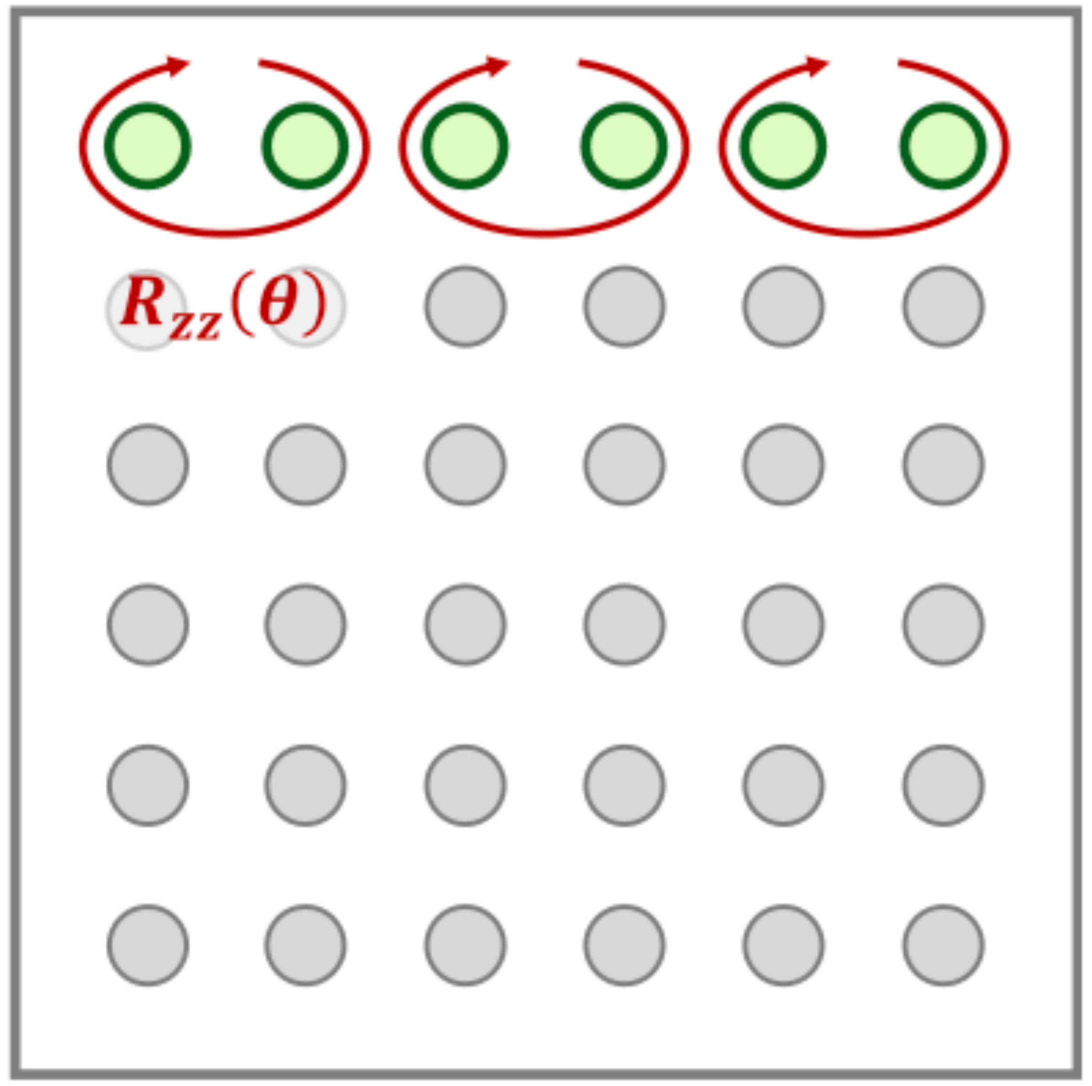
Inject using [[4,1,1,2]] and expand with error detection.

Choi's scheme



Choi et al. *"Fault tolerant non-clifford state preparation for arbitrary rotations."* arXiv:2303.17380.

Our improvement



Reducing logical error rate with increasing success probability arXiv:2408.14848

$$\prod_{i\in Q_z} \hat{R}_{z,i}(\theta)\,|+\rangle_L = \cos^d\theta\,|+\rangle_L + i^d\sin^d\theta\,|-\rangle_L + (Z\text{-error terms})$$

$$\boxed{\tilde{P}_L(\theta_*) = \alpha_{\mathrm{RUS}}|\theta_*|p_{\mathrm{ph}}.} \qquad \theta_*(\theta,d) \equiv \sin^{-1}\left(\frac{1}{\sqrt{p_{\mathrm{ideal}}}}\sin^d\theta\right)$$

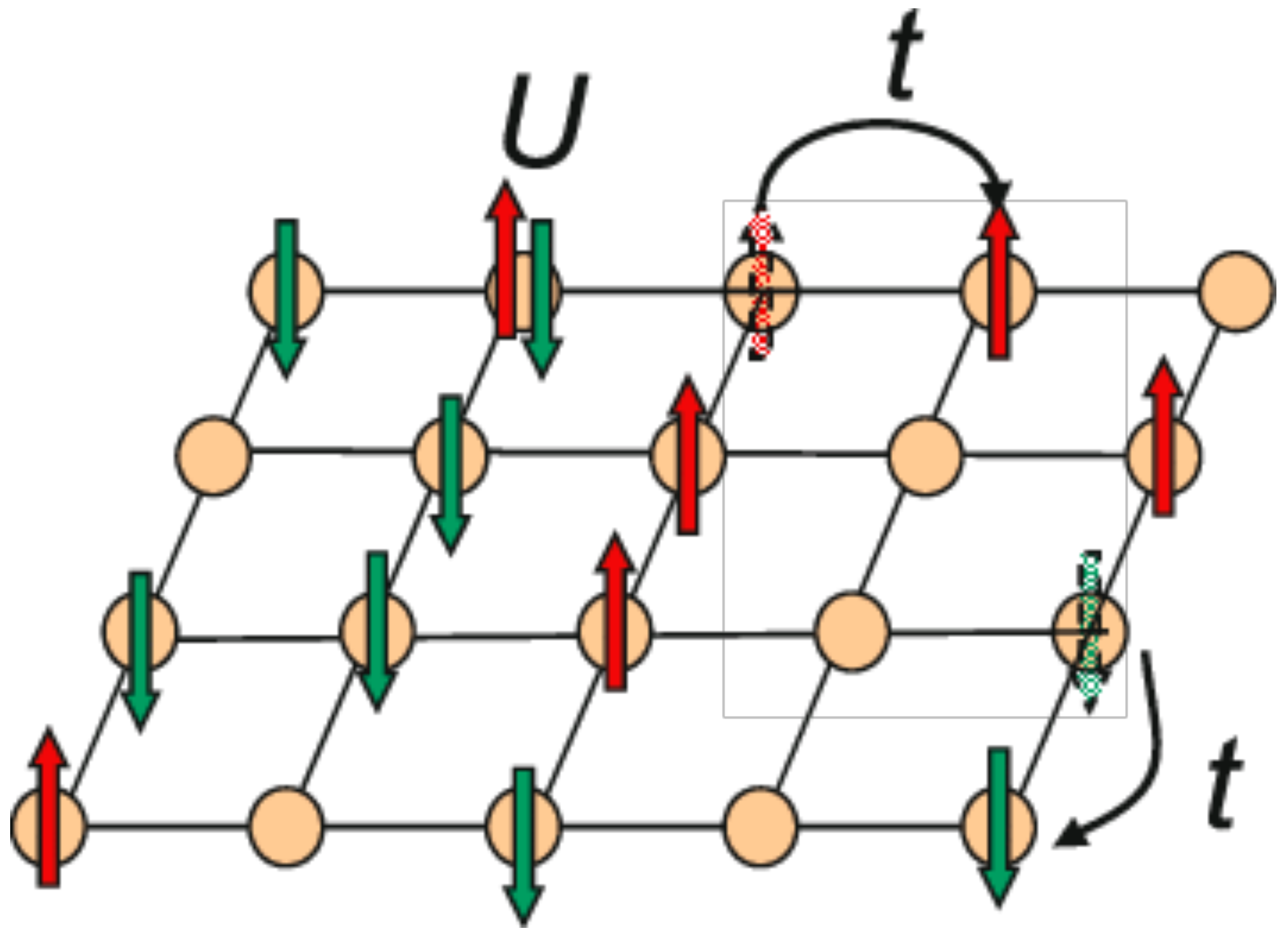
logical error rate is factored by the rotation angle

Resource estimation for QCELS

Toshio *et al.* "*Practical quantum advantage on partially fault-tolerant quantum computer.*" arXiv:2408.14848.

2D (*L*,*L*) site Hubbard model:

$$H = -\frac{t}{2}\sum_{\langle i,j\rangle,\sigma} \hat{c}^\dagger_{i\sigma}\hat{c}_{j\sigma} + \frac{U}{4}\sum_i \hat{n}_{i\uparrow}\hat{n}_{i\downarrow}$$



https://upload.wikimedia.org/wikipedia/commons/a/a8/2D-Hubbard-model.png

Required code distance:

$$p_L(p_{\mathrm{ph}}, d)^{-1} \geq 100 \times 4dLC_{\mathrm{av}}N_{\mathrm{max}}N_{\mathrm{patch}}.$$

d: code distance, *L*: lattice size, C_{av}: clocks for analog rotation gate

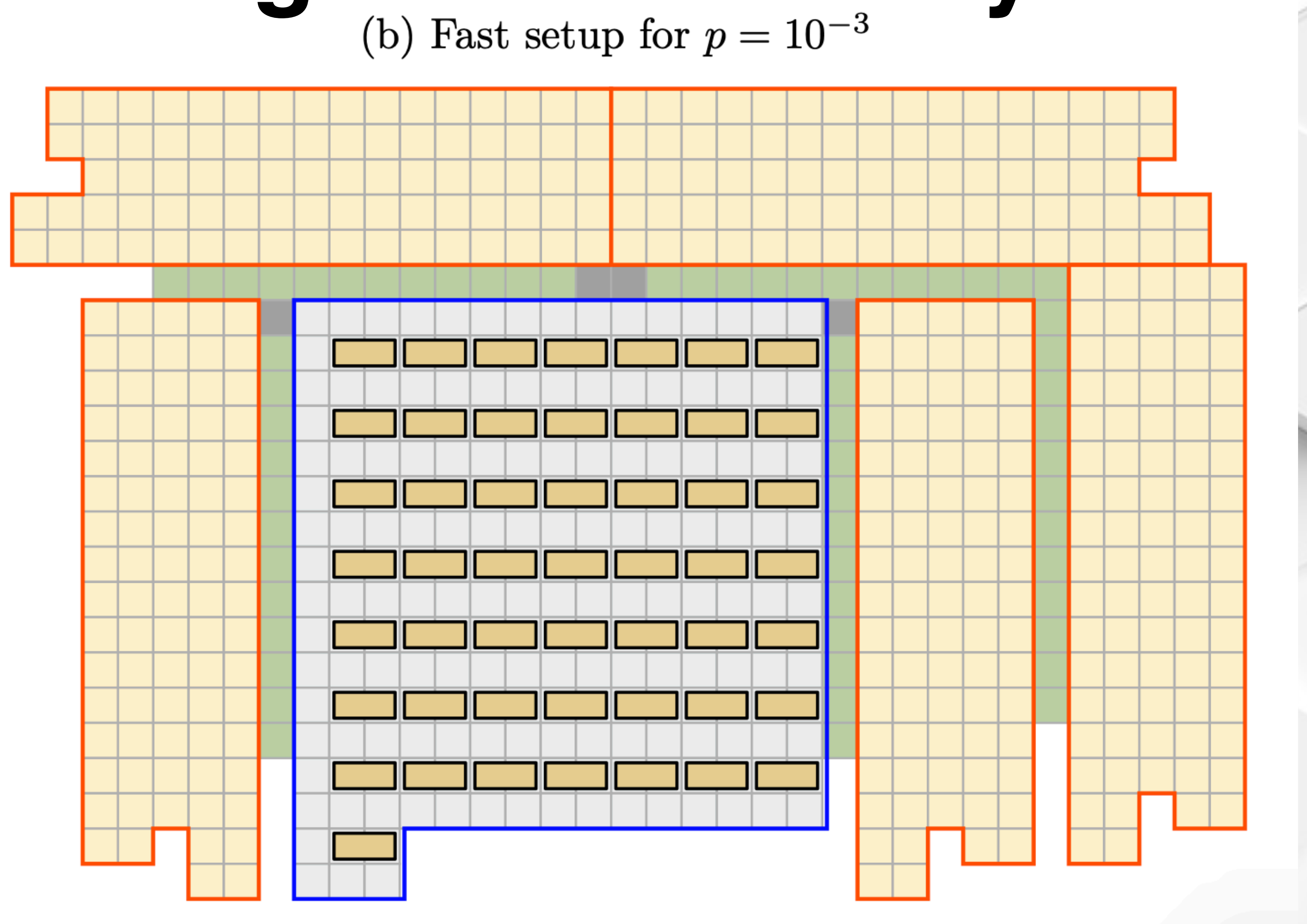
N_{patch} : # of logical qubit patches

# of Trotter steps for each Hadamard test: $N_{\mathrm{max}} \equiv \frac{T_{\mathrm{max}}}{2\Delta t} = \frac{\delta}{2\epsilon_{\mathrm{QPE}}}\sqrt{\frac{W}{\epsilon_{\mathrm{Trotter}}}}.$

Problem size		Code distance		Physical qubits		Execution time (hours)	
Lattice size	Data qubits	$p_{\mathrm{ph}} = 10^{-3}$	$p_{\mathrm{ph}} = 10^{-4}$	$p_{\mathrm{ph}} = 10^{-3}$	$p_{\mathrm{ph}} = 10^{-4}$	$p_{\mathrm{ph}} = 10^{-3}$	$p_{\mathrm{ph}} = 10^{-4}$
6×6	73	21	11	1.03e+05	2.83e+04	1.18e+03	7.88e+01
8×8	129	21	11	1.77e+05	4.86e+04	2.74e+04	2.15e+02
10×10	201	23	11	3.27e+05	7.48e+04	1.40e+06	5.08e+02

Reducing overhead for magic state distillation

magic state factory



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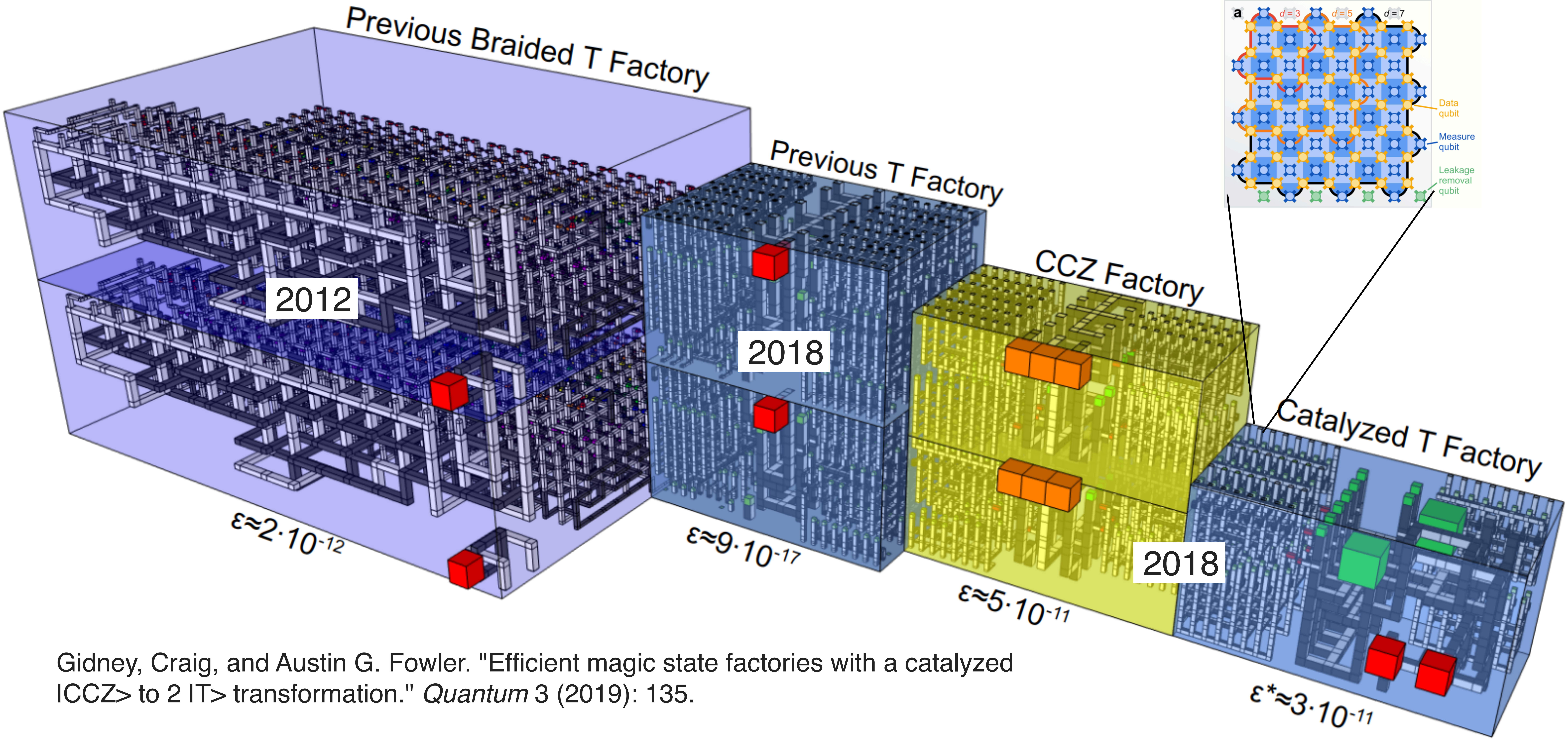
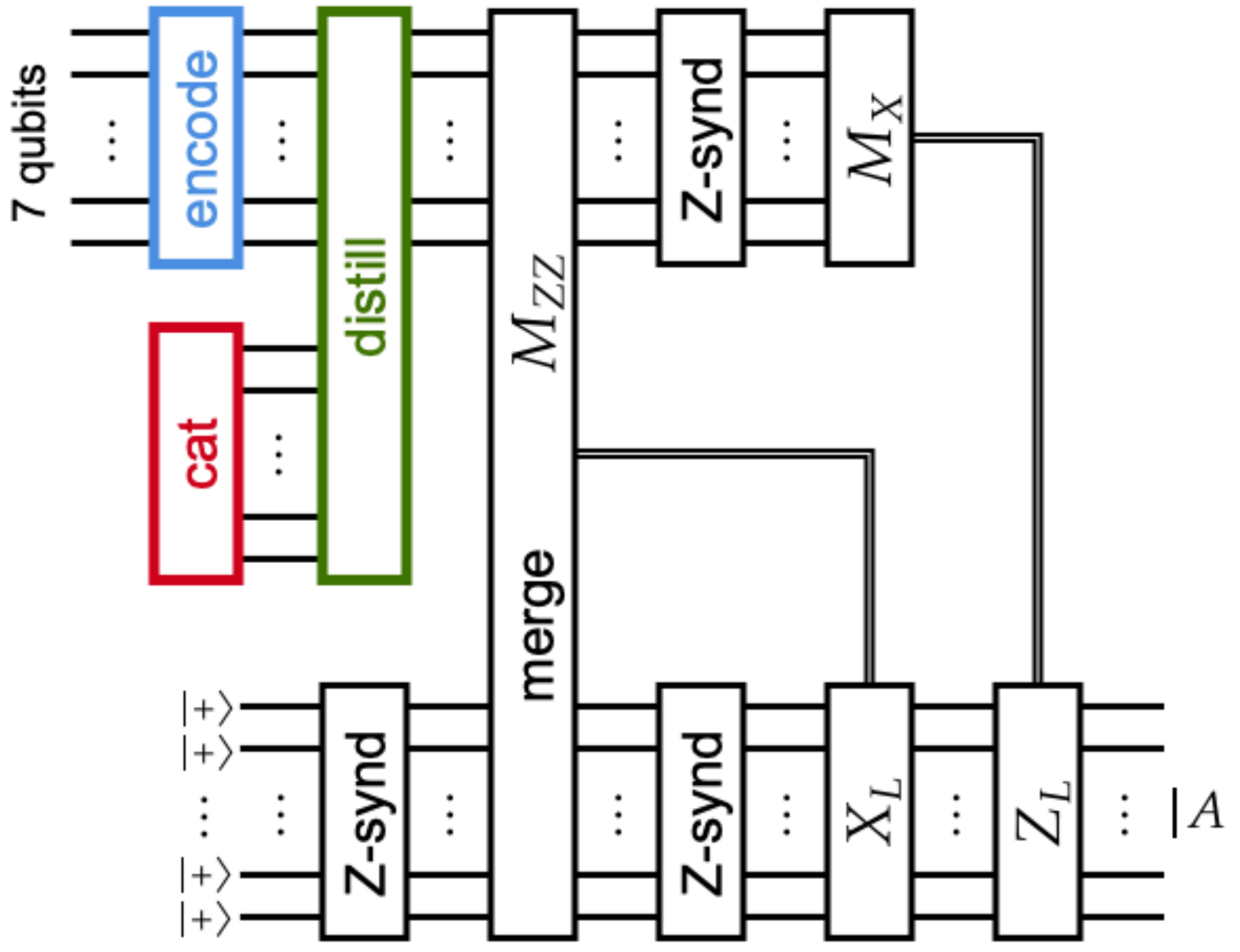


Gidney, Craig, and Austin G. Fowler. "Efficient magic state factories with a catalyzed |CCZ> to 2 |T> transformation." *Quantum* 3 (2019): 135.

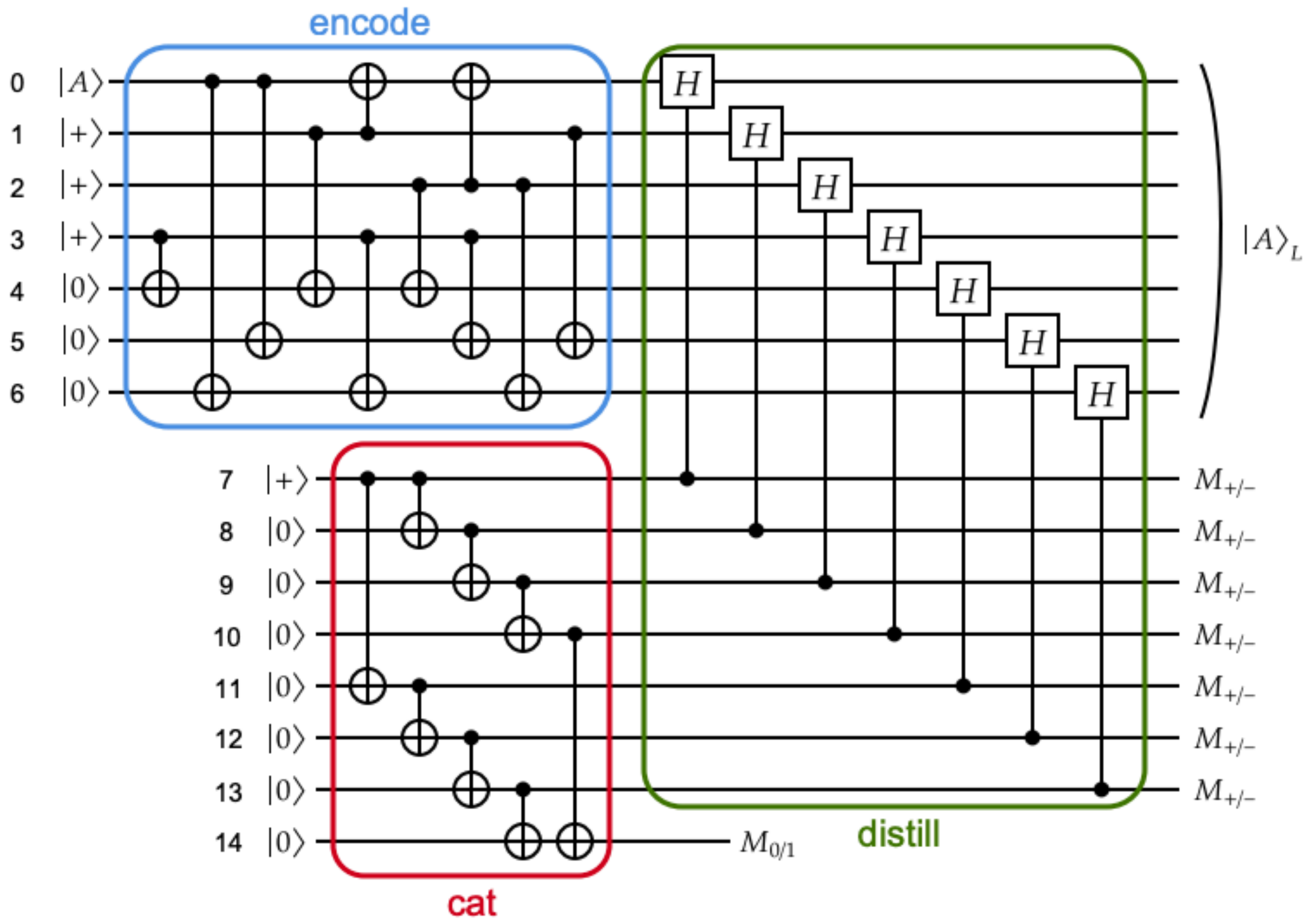
Zero Level Distillation: magic state distillation with physical qubits

T. Itogawa et al. "Even more efficient magic state distillation by zero-level distillation." arXiv:2403.03991 (2024).

Not logical but physical fault-tolerant circuit for magic state distillation:



Steane's 7-qubit code



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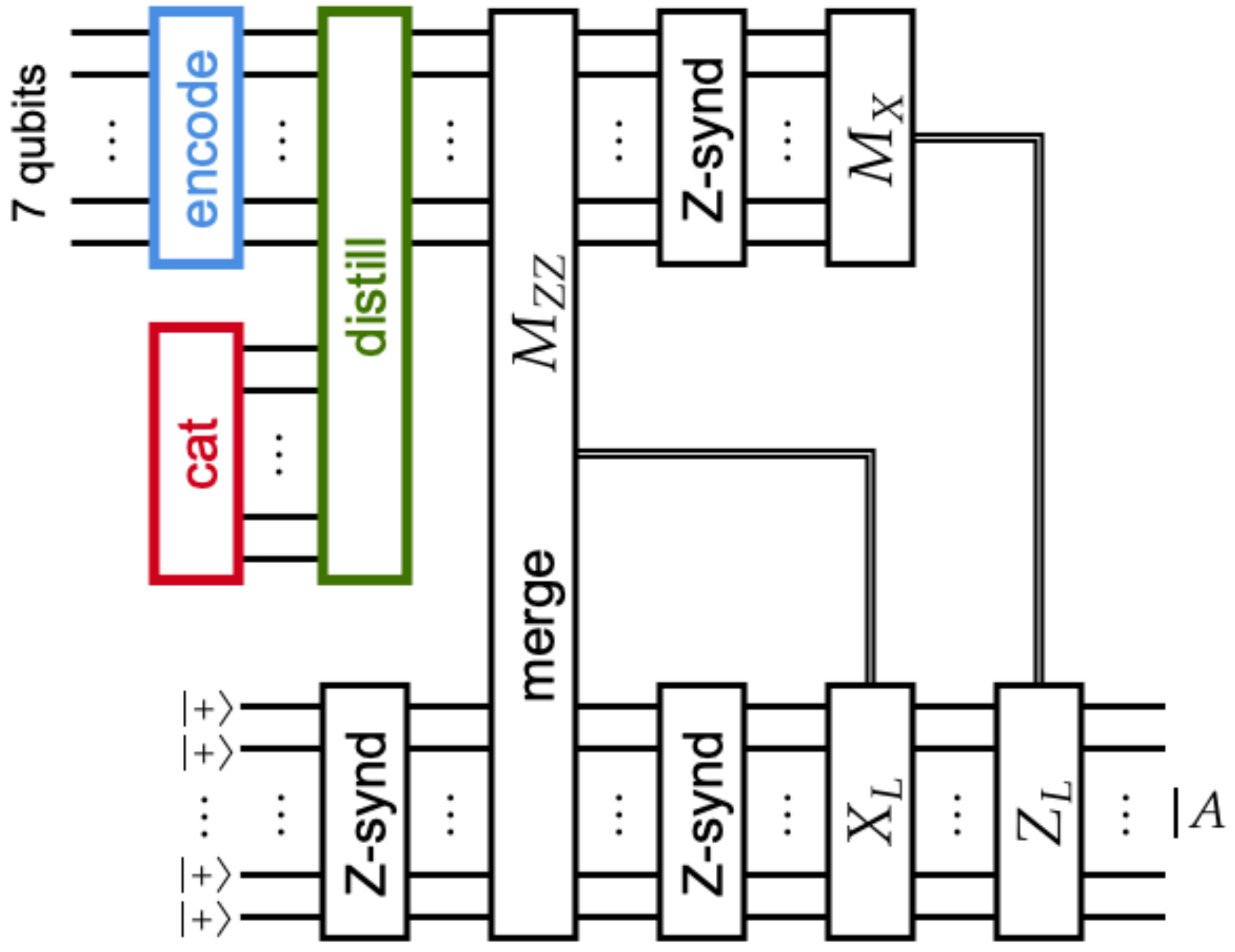

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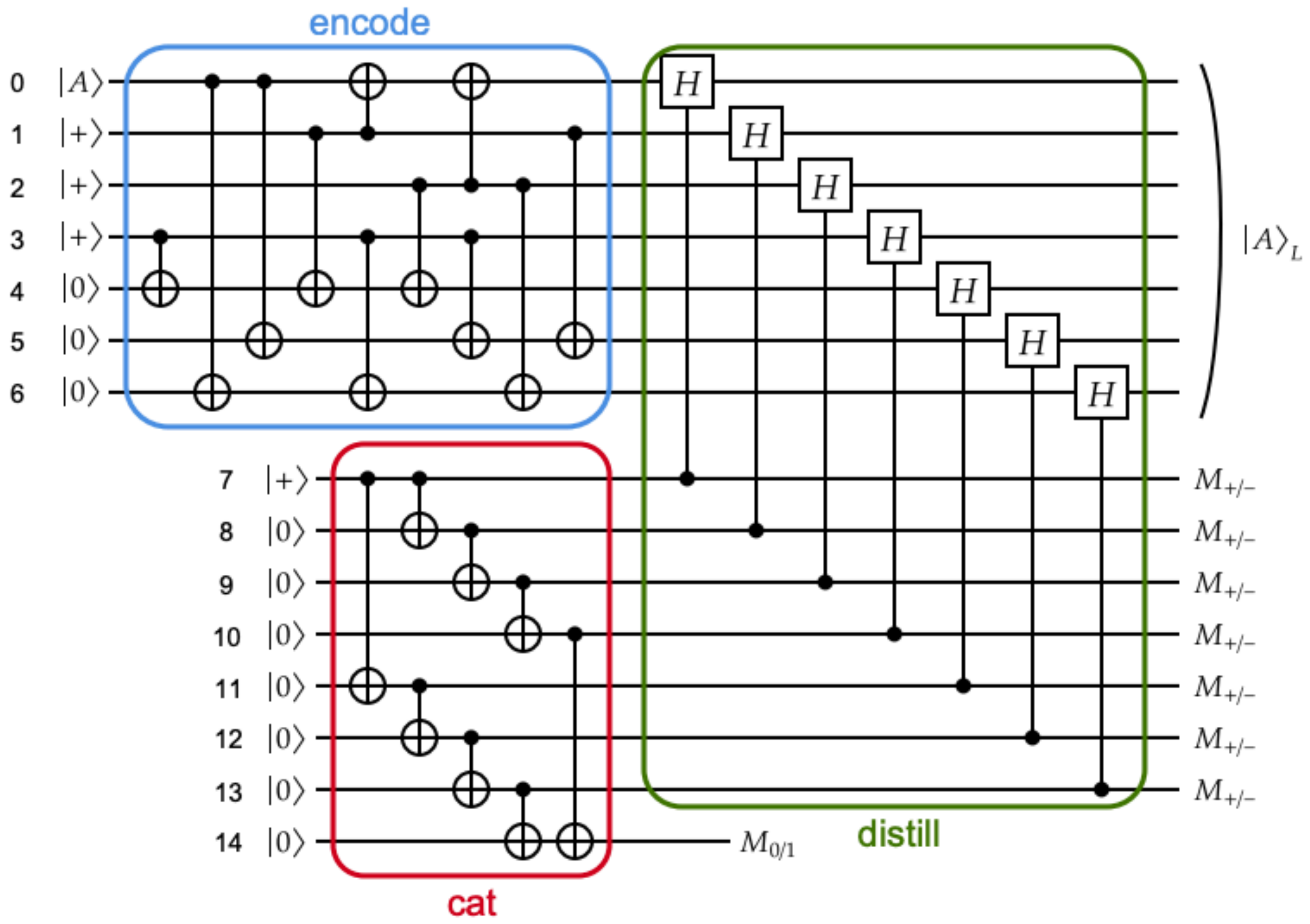
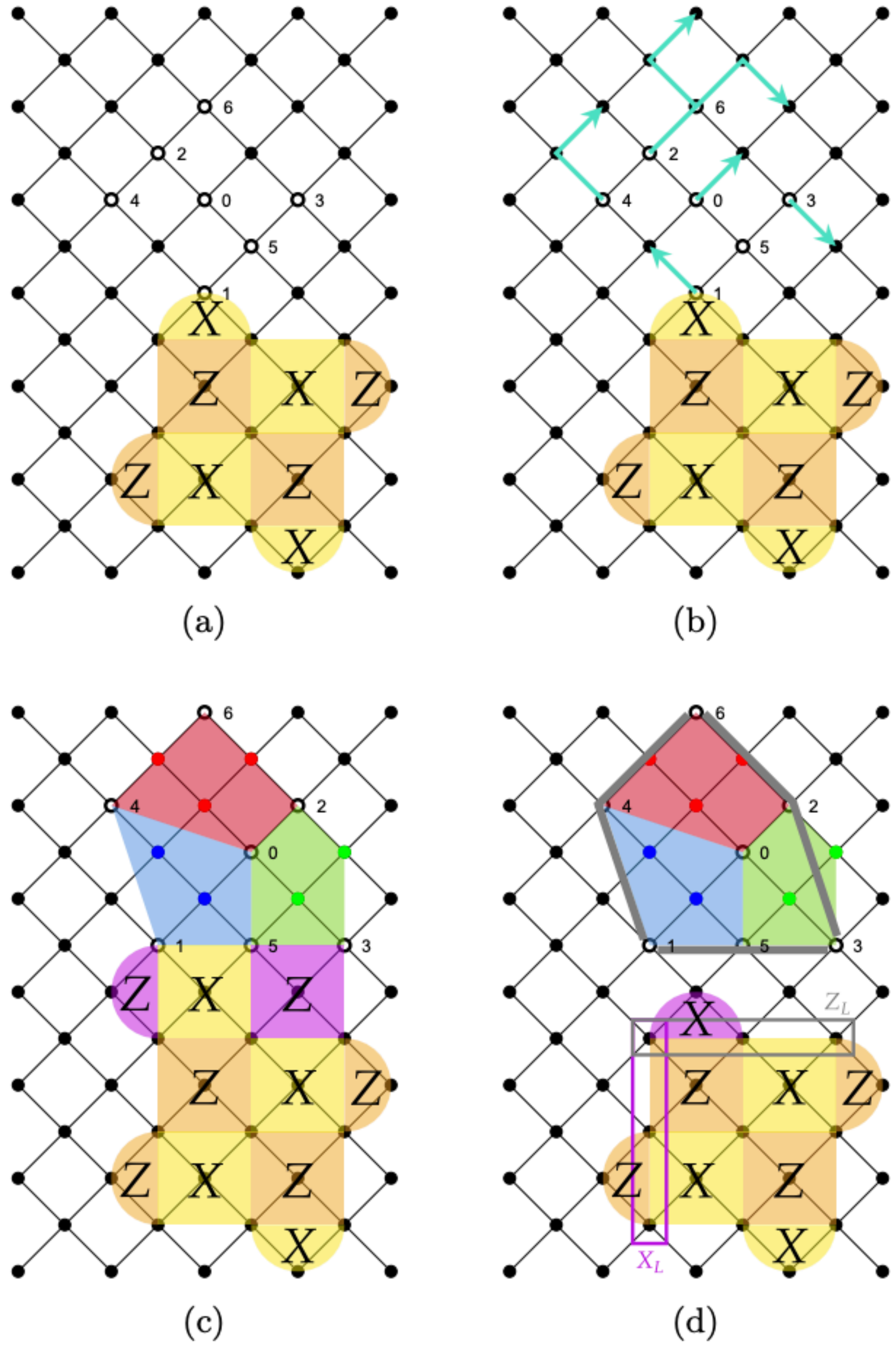


Implementation on square lattice architecture



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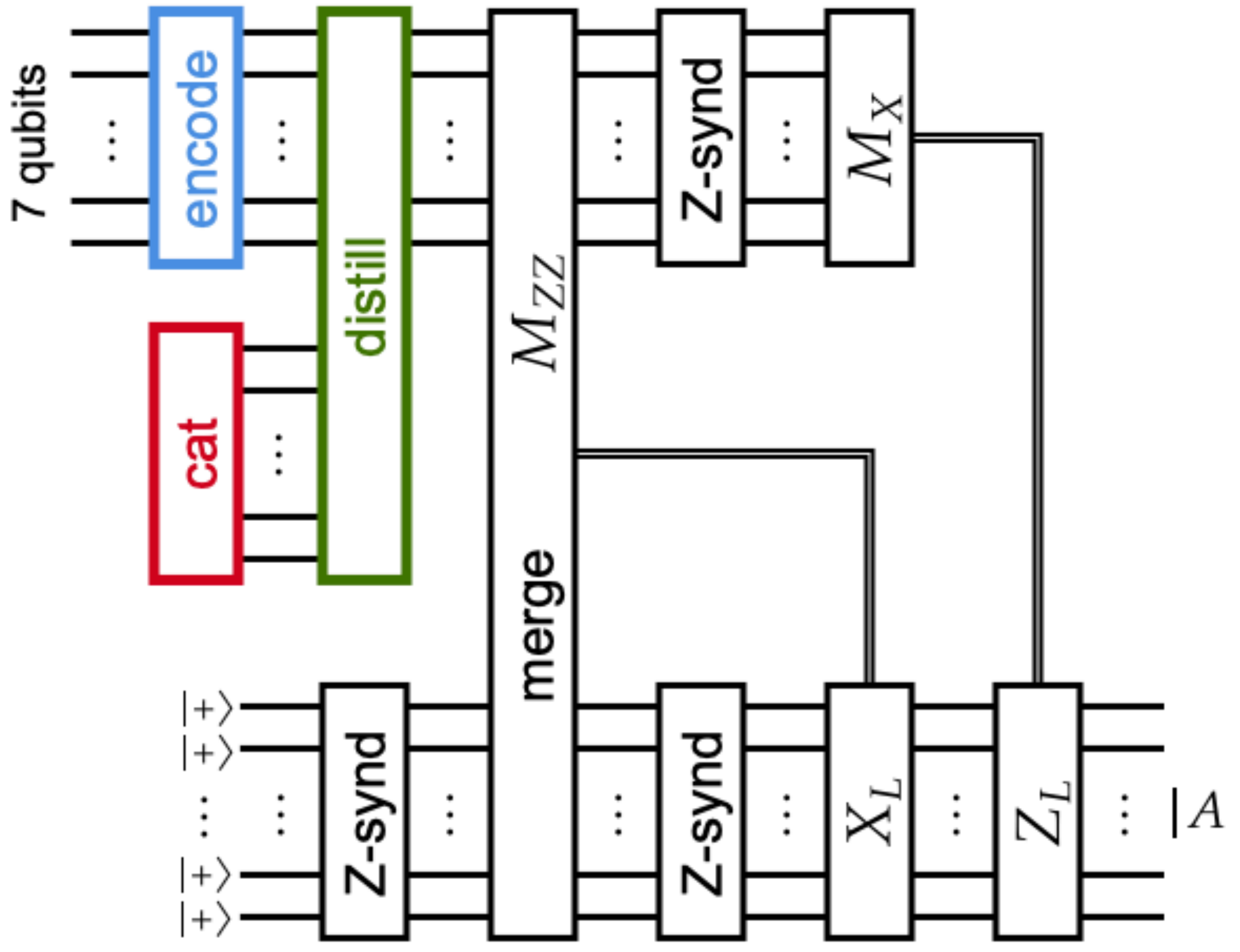

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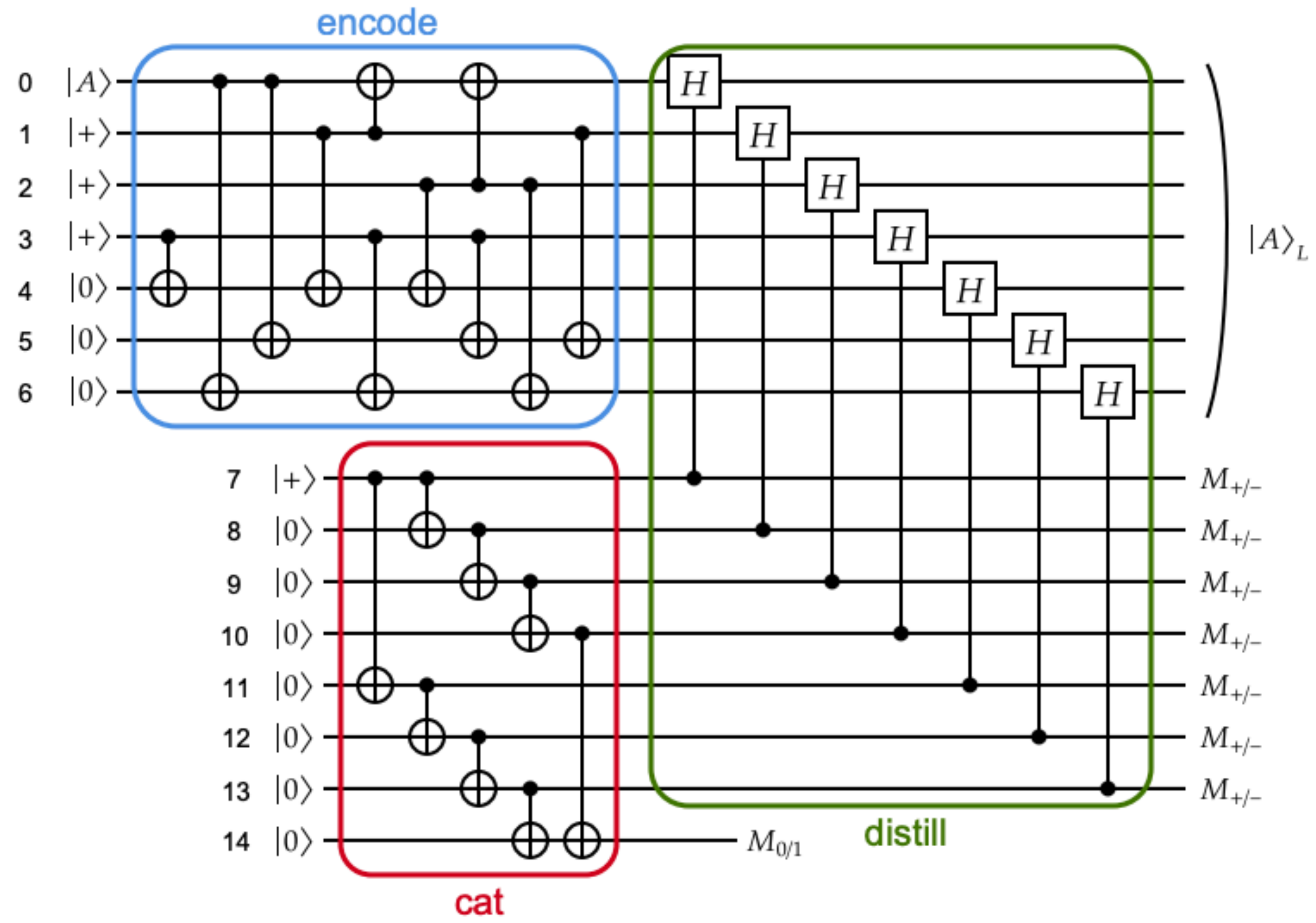

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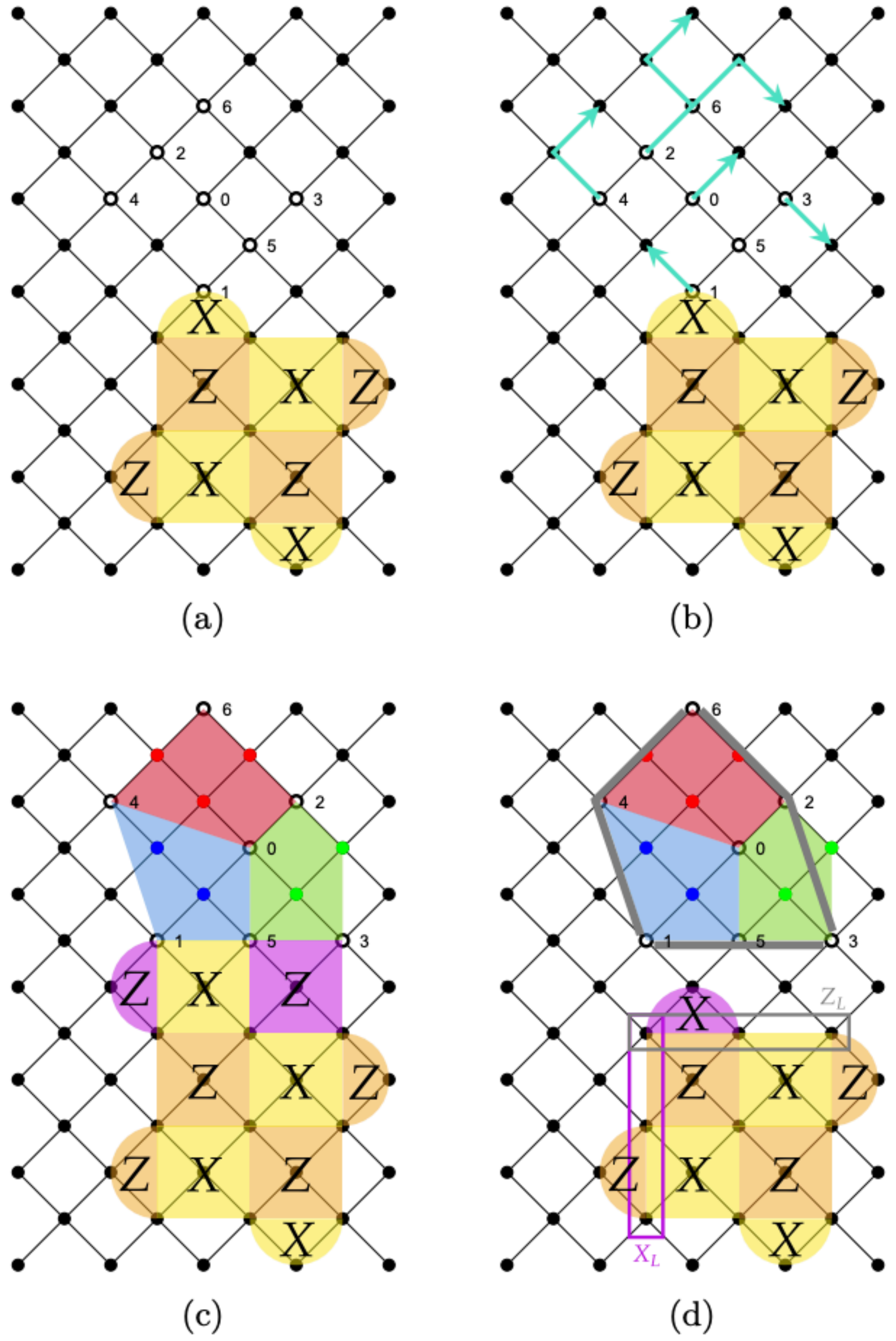
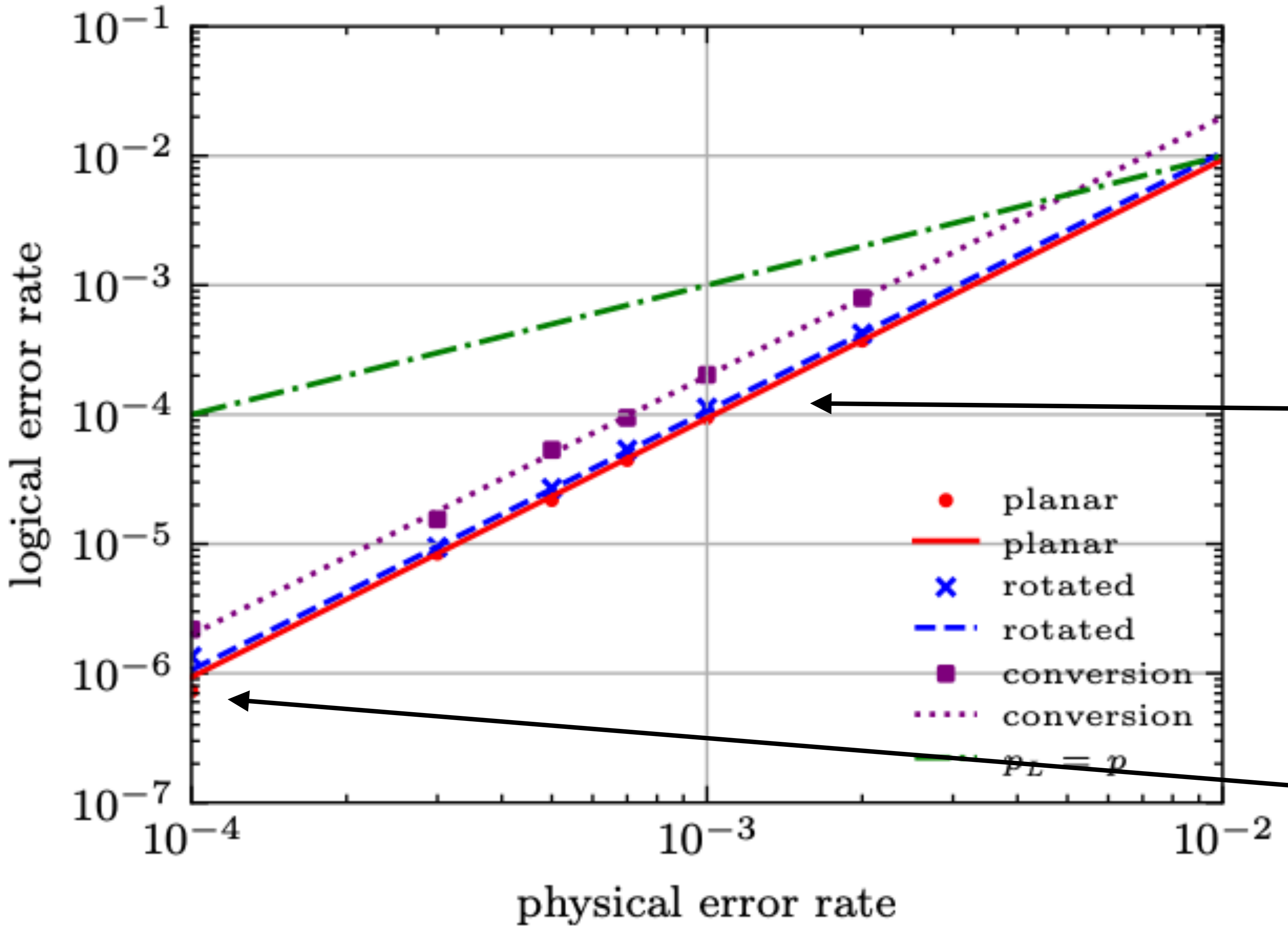


Phys vs Logical error rate



logical error ≃ 100p^2

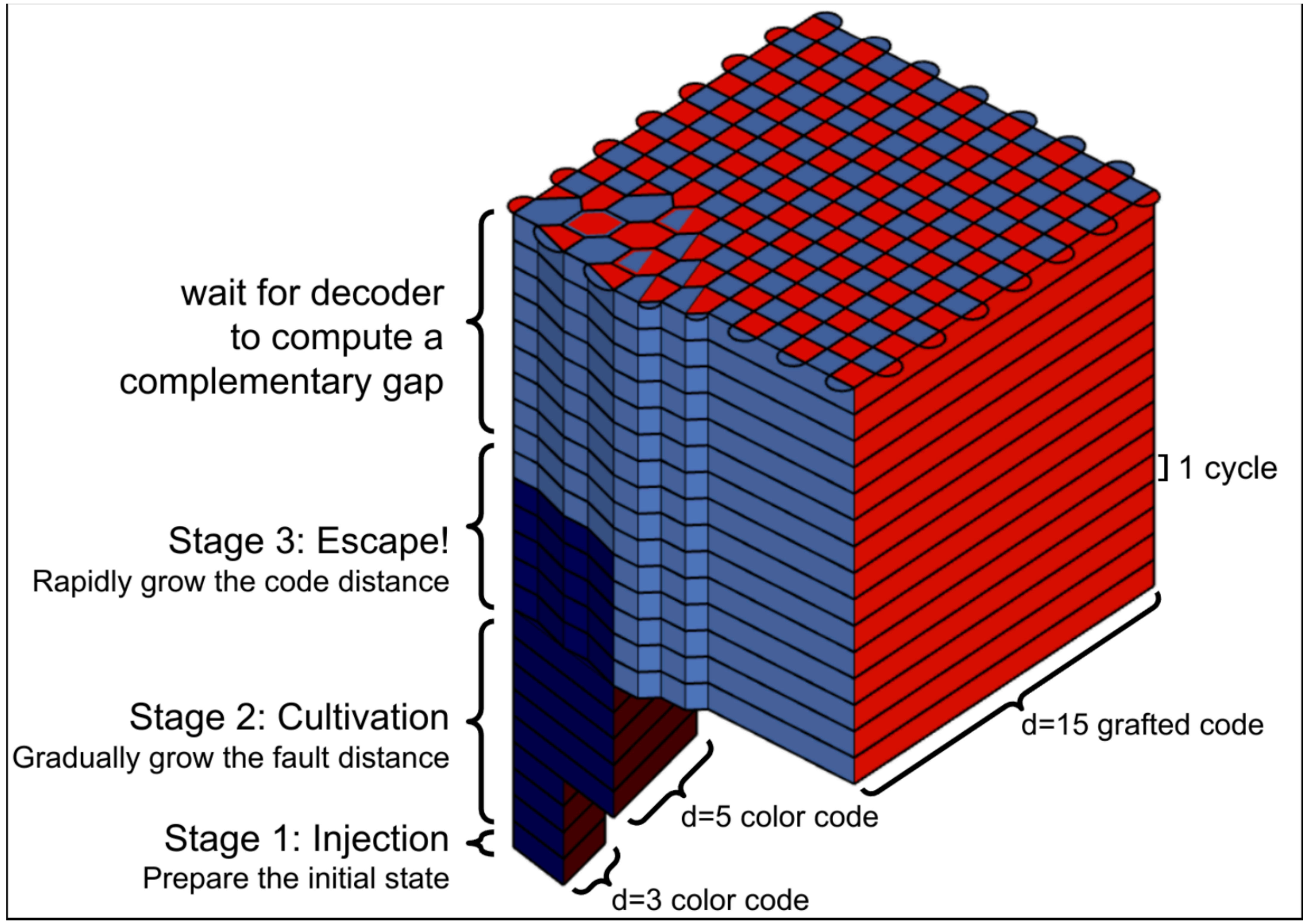
logical error 10^{-4}
(physical error rate 0.1%)

logical error 10^{-6}
(physical error rate 0.01%)

Magic State Cultivation

C. Gidney et al., "Magic state cultivation: growing T states as cheap as CNOT gates." arXiv:2409.17595.

Itogawa et al [Ito+24] improved on this by showing that the idea still works with a simple square grid connectivity and 10^{-3} uniform depolarizing circuit noise. These improvements are what brought the construction to our attention. In this paper we further refine the construction, enormously improving its performance. We make five main improvements.



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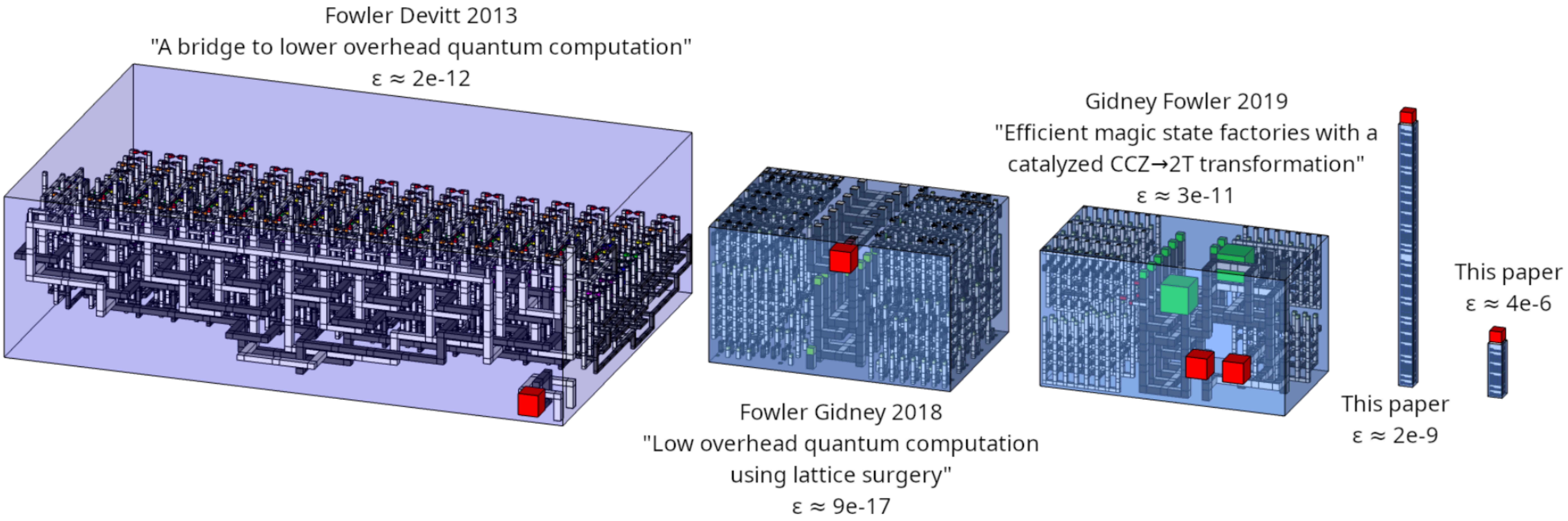

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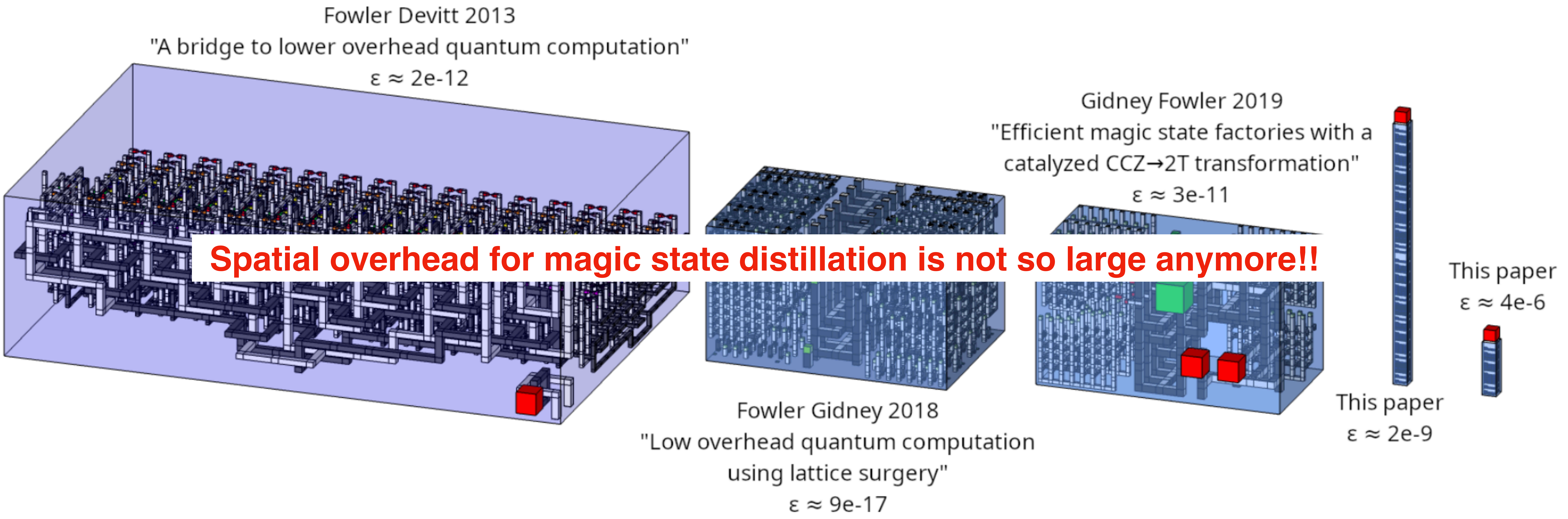

Collaboration with QunaSys

QUNASYS

We are Quantum Native.

Democratization of state-of-the-art research output

QURI SDK: FTQC R&D toolbox for domain experts

NEW

QURI SDK

Get time-to-market insights
based on real devices and architectures!

Application	QURI PARTS: Fundamental building blocks for QURI SDK	NEW QURI ALGO: Ready-to-use algorithm components/framework
Algorithm		
Quantum circuit (Optimization)		NEW QURI VM: Architecture/device abstraction Analysis/optimization/ simulation/execution
(E)FTQC architecture		
Hardware	(Partnering with hardware developers)	

- ✓ Modular
- ✓ Platform-agnostic
- ✓ High-performance

- ✓ Problem mapping to quantum algorithms
- ✓ Ready-to-use (E)FTQC algorithms as components

- ✓ Develop portable algorithms
- ✓ Evaluate performance and simulate on (E)FTQC arch
  - Fidelity/latency
  - Fast/noisy simulation

QURI SDK (documents)

QURI PARTS

For QC experts ⟷ For Domain Experts (Ex. Computational Chemistry) or just not doing everything from scratch

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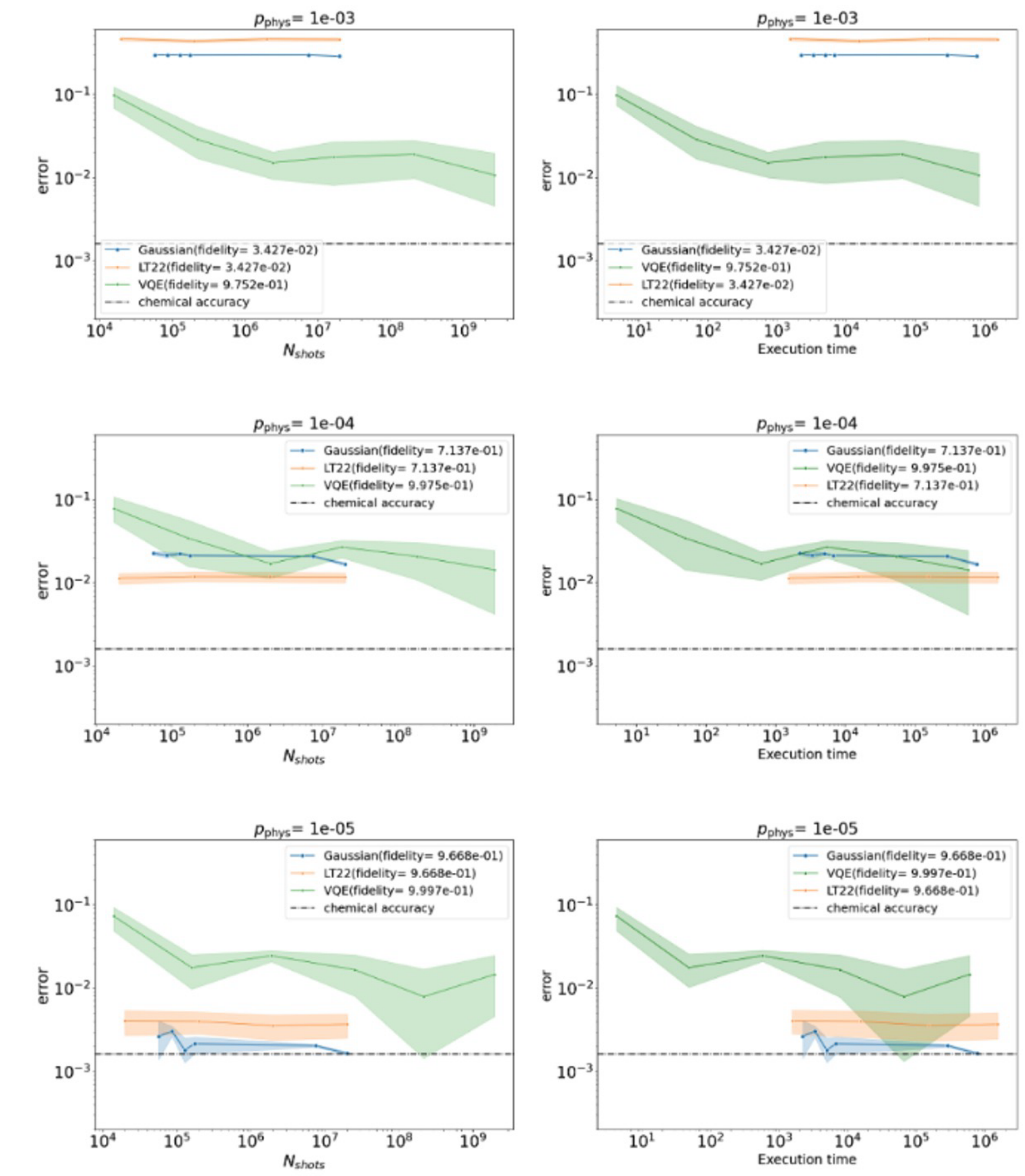
QURI ALGO - Ready-to-use algorithm components

- With implementations of known algorithm components provided, you can start running the algorithms from day one
    - Few lines of code needed
    - Problem setup is easy to understand
- General algorithms with classical pre-/post-processing
    - Statistical phase estimation (LT22/Gaussian filter)
    - QSCI (Quantum-selected configuration interaction)
- Problem-encoding into quantum circuits
    - Trotter time evolution circuit of Hamiltonian
    - Hadamard test with controlled time evolution
    - Quantum circuit compilation (QAQC/LVQC)

```
noisy_lt22 = SingleSignalLT22GSEE(
    state=state,
    cdf_parameters=rough_signal_param,
    post_processing_param=rough_post_process_param,
    time_evo_estimator=noisy_trotter_time_evolution_estimator,
    tau=tau,
)

noisy_lt22_energy = noisy_lt22()
print("Noisy LT22 energy:", noisy
print("Energy error:", noisy_lt2
pprint(noisy_lt22.resource)

Noisy LT22 energy: -1.07659396824322
Energy error: 0.0245563619893944 82
SPEGSEERecorder(max_evolution_time=4
                total_evolution_time
                n_shots=20000)
```



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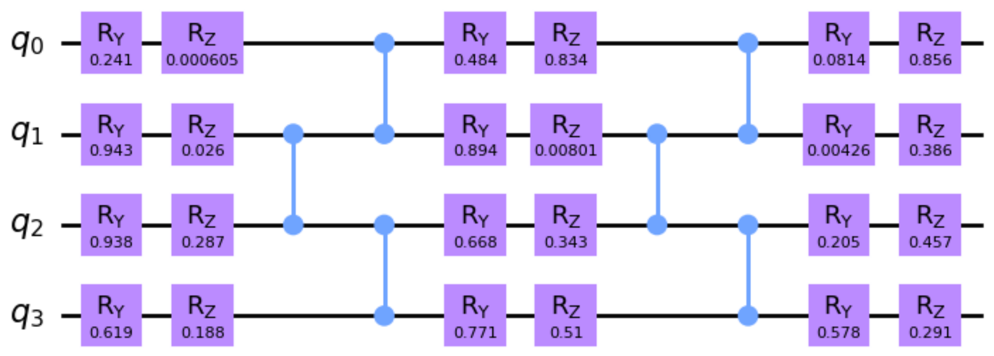
QURI VM : Logical circuit level performance evaluation

- With logical circuit level evaluation,
  - Characteristics of (early-)FTQC architecture can be incorporated as parameters
  - Performance on NISQ and (early-)FTQC device can be compared
- Developing the logical circuit level resource estimator targeted at early-FTQC devices
  - Target circuits: NISQ-like circuits for ~100 qubits
    - Chemistry-oriented state preparation
    - Condensed matter time evolution
  - Indicators: fidelity, execution time etc.
- How can we incorporate characteristics of various architectures and devices?

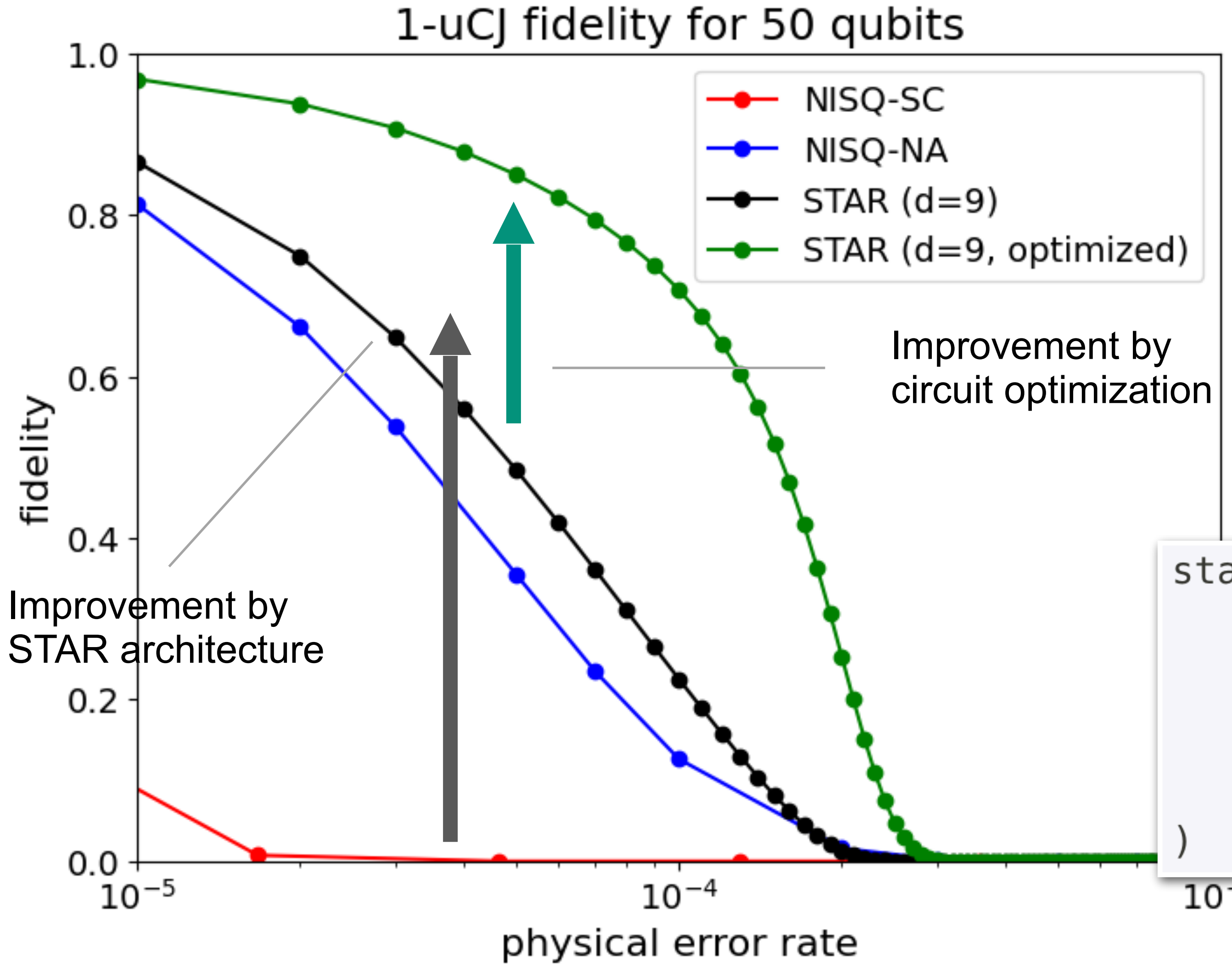


Logical circuit level resource estimator

NISQ device characteristics
Error rates / gate execution time / …

```
nisq_spcond_lattice.generate_device_property(
    lattice=SquareLattice(4, 4),
    native_gates=("RZ", "SqrtX", "X", "CNOT"),
)
```

```
nisq_iontrap_device.generate_device_property(
    qubit_count=16,
    native_gates=("RZ", "SqrtX", "X", "CNOT"),
    gate_error_1q=1.53e-3,
    gate_error_2q=4e-2,
    gate_error_meas=2.5e-3,
    gate_time_1q=TimeValue(10, TimeUnit.MICROSECOND),
    gate_time_2q=TimeValue(200, TimeUnit.MICROSECOND),
    gate_time_meas=TimeValue(130, TimeUnit.MICROSECOND),
)
```



EFTQC device/arch characteristics
Physical error rates / gate execution time / code distance / …

```
star_device.generate_device_property(
    qubit_count=50,
    code_distance=9,
    qec_cycle=TimeValue(value=1.0, unit=TimeUnit.MICROSECOND),
    physical_error_rate=1.0e-4,
)
```

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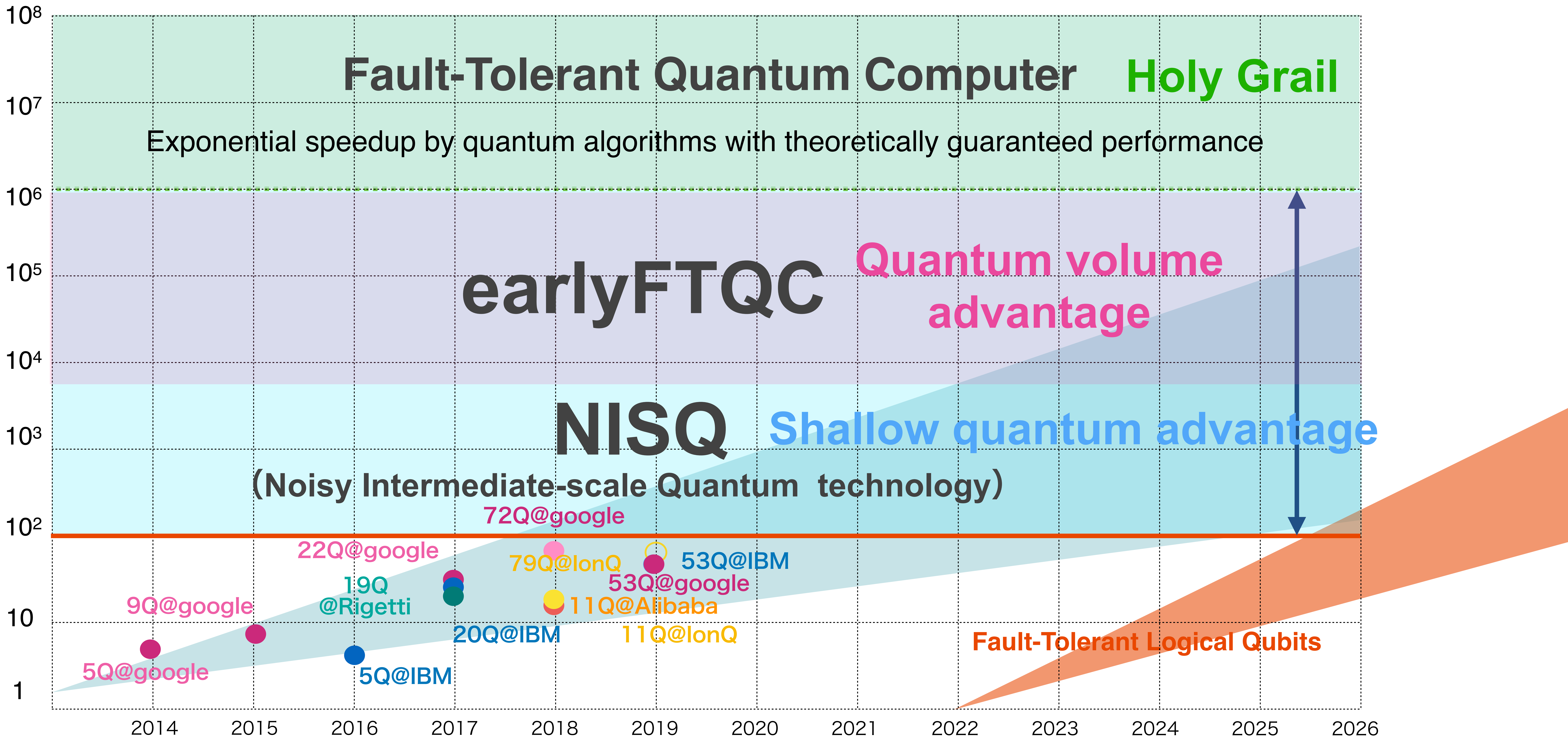
EarlyFTQC: Closing the gap between NISQ and FTQC



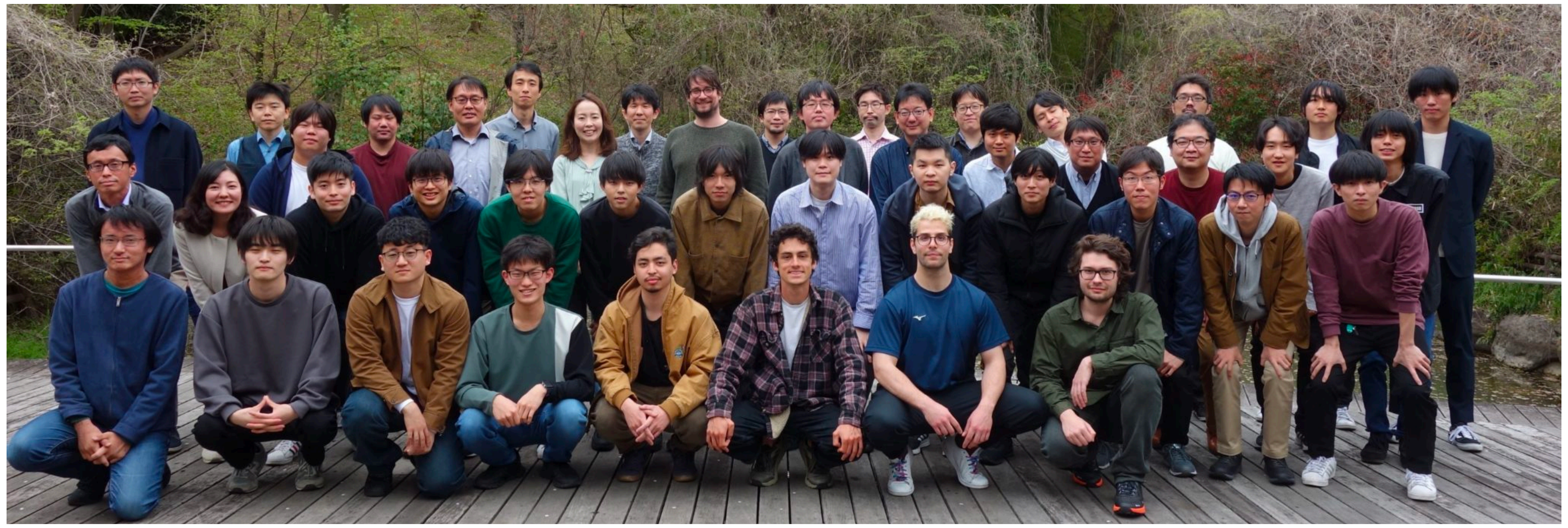
Summary

- Put another milestone between NISQ and FTQC.
- 10,000 qubits and 10^{-3}-10^{-4} error rate will provide a good opportunity to obtain QuantumVolume advantage as earlyFTQC.
- What scale of quantum algorithms are feasible?
- Can we add resource efficient magic state distillation?
  →Even more efficient magic state distillation by zero-level distillation
  T Itogawa, Y Takada, Y Hirano, K Fujii, arXiv preprint arXiv:2403.03991.
- Other hardware platform e.g. trapped ions, cold atoms?
  →Comming soon.

FUJITSU

Fujitsu Quantum Computing Joint Research Division at QIQB

- Akahoshi-Maruyama-Oshima-Sato-KF, PRX Quantum **5**, 010337 (2024)
- Toshio-Akahoshi-Fujisaki-Oshima-Sato-KF, arXiv:2408.14848
- Akahoshi-Toshio-Fujisaki-Oshima-Sato-KF, arXiv:2408.14929



大阪大学 OSAKA UNIVERSITY Quantum Computing (theory) Group at Osaka Univ.

QURI SDK QUNASYS We are Quantum Native.

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